ARTIKEL RISET

A Review Of The Linear Response Function In Condensed Matter Physics And Their Application In Some Elementary Processes

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Abstract

Linear response theory in quantum theory with its linear response function and its quantization process has been formulated. The relation between the linear response function with its generalized susceptibility, its symmetry properties, and its analyticity has been studied. These properties produce the dispersion relation or Kramers-Kronig relation. The explicit form of the quantum response function and generalized susceptibility also been reviewed. Applications of linear response functions have been described for three elementary processes. The process discussed is the magnetic field disturbance in the magnetic system that generates magnetic susceptibility, and the electric field disturbance in the electrical system that generates electrical conductivity tensor and the ferromagnet Heisenberg that generates its generalized susceptibility.

Keywords: linear response theory; response function; general susceptibility; magnetic susceptibility; electric conductivity tensor; Heisenberg ferromagnet

1 Introduction

Response functions describe the response of a material due to the external field. The external field can be in the form of an electric field and a magnetic field or others. An example of a response function that is well known is the susceptibility of matter, both due to the electric field or magnetic field. Electric susceptibility of an object illustrates the ease of object to be polarized in response to an electric field applied to it. Similar to the electric susceptibility, magnetic susceptibility is a measure of the ease of objects affected by the external magnetic field, and the influence of external fields generate magnetic dipole moment per unit volume in the field.

Electric susceptibility gives an overview of the electric polarization, and magnetic susceptibility objects give an idea of the formation of the magnetic dipole moment. Both describe the response of objects on the external field (either electric or magnetic) applied to the objects. The response of the material due to the external field determines the properties of objects that are very important to know. As an example, in the material that has a positive magnetic susceptibility (called paramagnetic material), the magnetic field inside the material will be gained by the magnetic dipole moment formed by an external magnetic field. On objects with negative susceptibility (called diamagnetic material), the reverse process occurs. The magnetic field inside the material will be weakened. This weakening is caused by the formation of magnetic dipole moments in the opposite direction of the external magnetic field.

Generally, the response function can provide an overview of the properties of the considered material. Therefore, knowledge about the response function is becoming essential in science and technology. Previously, research on the formulation of the response function is using the classical theory, without the use of quantum theory. As a result, the validity of the formulation of response function has a limited scope, only in classical cases. These limitations, for instance, are for materials in large quantities and large size only. Therefore, the formulation of response functions using quantum theory is needed to support the development of science and technology began to spread to those quantum cases. Thus, in this study will be redefined response function in quantum theory, along with its application to some elementary processes.

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The linear response function of a system due to external stimuli is associated with a Green's function, as stated by Rickayzen (1980)[1] and Cottam-Tiley (1989)[2]. In the Green's function was found that the correlation function has a dominant role. It can be said that the Green's function is a generalization that is appropriate for the concept of correlation functions. The correlation function is a function of the distribution in statistical mechanics, which describes the relationship between the two operators[3]. Therefore, the response function of the system can be determined if the correlation function can be determined.

The formulation of the linear response function involves the Green's function, and the correlation function is given the prominence of the method that is easy and simple, even though for many-particle systems. Green's function contains complete and sufficient information about the many-particle systems. Calculations involving correlation function involving the boundary conditions can be facilitated using the spectral theorem[3].

Some essential applications of the linear response theory include the formulation of a common susceptibility to the quantum system, as has been stated by Jensen-Mackintosh (1991)[4] in his book on rare earth magnetism. In the same book, Jensen-Mackintosh also gives another example that is applied to Heisenberg ferromagnet. Another vital application is the study of spin waves in objects antiferromagnet and ferrimagnet[5]. We also discuss other applications of the theory of linear function in magnetic susceptibility, electrical conductivity[3], and linear response in the Heisenberg ferromagnet.

2 Basic Definition and Properties Of Linear Response Function

In general, a physical system will respond if stimulus or interference is given. The higher the stimulus is given, the system response is higher. This is called a **linear response**. Most physical systems fall into this category, although this applies only to relatively small disturbances. If the given interference is large enough, the response produced by the system is no longer linear. The response also depends on the time behavior of the stimuli. For example, if the stimulus is in the form of an oscillating voltage (AC voltage), then the system's response will be different when compared to systems stimulated with DC voltage. The response system will depend on the time dependency of the stimulus, which in general is an arbitrary function of time. This problem will be more easily analyzed if the initial stimulus is broken down according to its frequency components using Fourier transform in Fourier analysis. Each component of this stimulus will be sought for the response. Then these responses are summed (superposition principle applies considering linear response). This is the basis of reasoning that will be used.

Consider the system response is written as $\mathcal{R}(t)$, and stimulation or "forces" as $f(t)$ which is given to the system. The assumptions are used:

1. The system will start to respond if the stimulus started. In this case, causality applied. Responses should not precede stimulation.
2. The response is linear, so it applies the principle of superposition.

The general relationship between responses at time t to the stimuli $f(t')$ which is working from time $t = -\infty$ to t is in the form of a superposition of $f(t')$

$$\mathcal{R}(t) = \int_{-\infty}^{t} \phi(t,t') f(t') dt' \tag{1}$$

with $\phi(t,t')$ is the response per unit stimulus that is called as **response function**. Equation (1) shows that the response at time t can only be caused by previous stimulation. Therefore, this response is called a causal response. Stimulation at a time greater than t, will not give any influence to the current response $\mathcal{R}(t)$.

A change needs to be made to our equation, considering the response does not depend on absolute time, but only depends on the time difference between the stimulus and the response measurements, so $\phi(t,t') = \phi(t-t;)$ and

$$\mathcal{R}(t) = \int_{-\infty}^{t} \phi(t-t') f(t') dt' \tag{2}$$

The form of the response function $\phi(t-t')$ will conform with the stimulus and the response. If they are scalar, then the response function is a scalar function. If they are vectors, then the response function is a 2-degree tensor. In general, if the stimulus and the response is an n-degree tensor, then the response function is a 2n-degree tensor.

Response function should behave as a decaying function with time. If the new force just worked, then the response was great, at a later time after the force worked, the response will be decreased and became more stable. A response function can oscillate, or at some time have negative value, as long as the function shape is progressively decayed towards zero or a specific value. Clearly, a response function also should not increase over time if no more force applied.

2.1 Generalized Susceptibility

The response equation in Eq. (2) can be transform to frequancy domain using standard Fourier transform

$$\int_{-\infty}^{\infty} e^{i\omega t}\bar{\mathcal{R}}(\omega)d\omega = \int_{-\infty}^{t} \phi(t-t') \int_{-\infty}^{\infty} e^{i\omega t'}\bar{f}(\omega)d\omega dt' \tag{3}$$

Then by changing the order of integration and introducing $\tau = t - t'$ we get

$$\tilde{\mathcal{R}}(\omega) = \left(\int_{0}^{\infty} e^{i\omega\tau}\phi(\tau)d\tau\right)\tilde{f}(\omega), \quad \forall\ \omega \in \Re \tag{4}$$

with $\Re$ is set of real number and by defining

$$\tilde{\mathcal{R}}(\omega) \equiv \chi(\omega)\tilde{f}(\omega), \quad \forall\ \omega \in \Re \tag{5}$$

This $\chi(\omega)$ is called as **generalized susceptibility**. This term is analogous with electric or magnetic susceptibility, which is a measure of the response of a material in an external field.

2.2 The Symmetry and Analytic Properties of Generalized Susceptibility

This is the equation that relates generalized susceptibility to its response function:

$$\chi(\omega) = \int_{0}^{\infty} e^{i\omega\tau}\phi(\tau)d\tau \quad \text{with } \tau \geq 0. \tag{6}$$

This equation is also called the Fourier-Laplace transform of the response function. Function $\phi(\tau)$ is a real function, so in general $\chi(\omega)$ is a complex-valued function. The equation above can be seen that

$$\text{Re}\ \chi(-\omega) = \text{Re}\ \chi(\omega), \text{ and} \tag{7}$$

$$\text{Im}\ \chi(-\omega) = -\text{Im}\ \chi(\omega) \tag{8}$$

for real ω. This shows that generalized susceptibility is symmetric.

The function $\chi(\omega)$ is an analytic function in the upper half-plane of the complex plane, as well as the real line (based on the initial assumption). Thus, for any closed loop in the upper half-plane ω, the integral contour obey

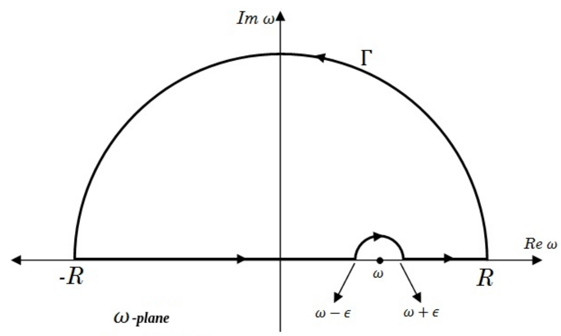
$$\oint_C \chi(\omega)d\omega = 0 \quad \text{(Cauchy's Integral Theorem)}. \tag{9}$$

2.3 The Dispersion Relation

Analytic properties of $\chi(\omega)$ permit the formulation $\chi(\omega')$(with ω' is a real physical value) that can be expressed in another real number ω, and this is called as a dispersion relation. We start by defining

$$f(\omega') = \frac{\chi(\omega')}{\omega' - \omega} \tag{10}$$

The equation has a simple pole on the real axis in ω-plane.



Gambar 1: Contour integration on ω-plane.

The function in Eq. (10) have contour shape of the as shown in Fig. 1, and by applying the Cauchy Integral theorem, then we have

$$\chi(\omega) = \frac{P}{i\pi}\int_{-\infty}^{\infty}\frac{\chi(\omega')d\omega'}{\omega' - \omega} = \frac{-iP}{\pi}\int_{-\infty}^{\infty}\frac{\chi(\omega')d\omega'}{\omega' - \omega} \tag{11}$$

which is an integration of all real ω' from minus infinity towards infinity. If the real and imaginary components are separated, it become

$$\text{Re}\ \chi(\omega) = \frac{P}{\pi}\int_{-\infty}^{\infty}\frac{\text{Im}\ \chi(\omega')d\omega'}{\omega' - \omega} \tag{12}$$

and

$$\text{Im}\ \chi(\omega) = -\frac{P}{\pi}\int_{-\infty}^{\infty}\frac{\text{Re}\ \chi(\omega')d\omega'}{\omega' - \omega}. \tag{13}$$

This form of both equations is called a **Hilbert transform pair**.

Equation (12) can be rewritten into

$$\text{Re}\ \chi(\omega) = \frac{2P}{\pi}\int_{0}^{\infty}\frac{d\omega'\ \omega'\ \text{Im}\ \chi(\omega')}{\omega'^2 - \omega^2}. \tag{14}$$

Equation (13) can be written in a similar way

$$\text{Im}\ \chi(\omega) = -\frac{2P\omega}{\pi}\int_{0}^{\infty}\frac{d\omega'\ \text{Re}\ \chi(\omega')}{\omega'^2 - \omega^2}. \tag{15}$$

Both of equations is called dispersion relation or also called Kramers-Kronig relation.

2.4 Changing from Classical Linear Response Theory toward Quantum Mechanics

Changing from the classical to the quantum theory can begin by reviewing a classical system and its response function, followed by the first quantization to obtain quantum theory. This discussion refers to Kubo (1957)[6].

In a closed system, the system Hamiltonian is written as $\mathcal{H}$. Dynamical motion of a system determined by $\mathcal{H}$ is called a natural motion. Consider an external force imposed on the system. Its energy can represent the impact of this perturbation,

$$\mathcal{H}'(t) = -AF(t). \tag{16}$$

In this discussion, the perturbation is limited to a weak force only. The response system will be sought at the linear rate approach. System response is in the changes the physical quantities $\Delta B(t)$ from its original B. Next, $\Delta B(t)$ will be stated in terms of the natural motion of the system.

At first, consider the classical mechanics perspective. Suppose the system is in a statistical ensemble represented by the distribution function $f(p,q)$ in phase space. The equations of motion describe the natural motion of the system is

$$\frac{\partial f}{\partial t} = -\sum\left(\frac{\partial f}{\partial q}\frac{\partial \mathcal{H}}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial \mathcal{H}}{\partial q}\right) = (\mathcal{H}, f) \tag{17}$$

with p and q is the set of canonical momentum and canonical coordinate, while the parenthesis is the Poisson bracket

$$(A,B) = \sum\left(\frac{\partial A}{\partial q}\frac{\partial B}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial B}{\partial q}\right). \tag{18}$$

Assume the distribution function is f at $t = -\infty$. Suppose that system is in a state of equilibrium, so $(\mathcal{H}, f) = 0$. Then disturbance in Eq. (16) is given to the system adiabatically at $t = -\infty$(with $F(t = -\infty) = 0$). The distribution function satisfies

$$\frac{\partial f'(t)}{\partial t} = (\mathcal{H}, f') + (\mathcal{H}'(t), f). \tag{19}$$

a linear approach causes

$$f'(t) = f + \Delta f$$

which can replace (19) with

$$\frac{\partial \Delta f}{\partial t} = (\mathcal{H}, \Delta f) - F(t)(A, f). \tag{20}$$

The solution is

$$\Delta f(t) = -\int_{-\infty}^{t} e^{i(t-t')L}(A,f)F(t')dt', \tag{21}$$

where the operator L is defined as an operation

$$iLg \equiv (\mathcal{H}, g). \tag{22}$$

So the change in $\Delta B(t)$ to a dynamic quantity B is given statistically by

$$\begin{aligned}\Delta B(t) &= \int \Delta f(t)\cdot B(p,q)d\Gamma \\ &= -\int d\Gamma \int_{-\infty}^{t}\left\{e^{i(t-t')L}(A,f)\right\}\cdot F(t')B(dt') \\ &= -\int d\Gamma \int_{-\infty}^{t}(A,f)B(t-t')F(t')dt'\end{aligned} \tag{23}$$

where $d\Gamma$ is the volume element in the phase space[7]. The last equation is obtained from the second equation by remembering that the transformation of e^{iLt} is a natural motion that conserves the size of the phase space and $B(t)$ is the dynamic motion of the phase function $B(p,q)$ that satisfies the equation

$$\dot{B}(p,q) = (B, \mathcal{H}).$$

The above equation is related to the Heisenberg equation of motion in quantum mechanics. The equation (23) means that the response $\Delta B(t)$ is a superposition of the impact of the stimulus $F(t')dt'$, for $-\infty < t' < t$. The response for each stimulus unit is referred to as **response function** or *after-effect function* $\phi_{BA}(t)$. The form of this response function can be known from Equation (23)

$$\phi_{AB}(t) = -\int d\Gamma(A,f)B(t) \tag{24}$$

which explains the response of ΔB when t after the stimulus is given. The response $\Delta B(t)$ to the equation (23) is written as

$$\Delta B(t) = \int_{-\infty}^{t}\phi_{AB}(t-t')F(t')dt'. \tag{25}$$

The above review applies even to the very sharp initial distribution, as long as the disturbance is quite small. Applied response functions are needed for macro-scale systems, which in these cases, the statistical average requires real meaning. If $\hat{A}$ and $\hat{B}$ are both macro-scale quantities, then the average ensemble $\Delta\hat{B}(t)$ can be observed. It is because a macro-scale system can be considered to consist of smaller systems so that the observed value of $\Delta\hat{B}$ is the number of components, and the change is relatively minimal.

The change to quantum mechanics can also be done, based on the classic formulation above. The distribution function in the classical phase space is then replaced by the ρ density operator matrix. The original ensemble that statistically represents the initial state of the system is determined by this density operator and satisfies $[\hat{\mathcal{H}}, \rho] = 0$, while the ensemble dynamic is affected by interference (16) is represented by $\rho'(t)$, which satisfies the equation

$$\frac{d}{dt}\rho'(t) = \frac{1}{i\hbar}[\hat{\mathcal{H}} + \hat{\mathcal{H}}'(t), \rho'(t)]. \tag{26}$$

The initial conditions are

$$\rho'(-\infty) = \rho$$

and $\rho'(t)$ are listed as

$$\rho'(t) = \rho + \Delta\rho(t).$$

The same steps as (20) direct the results to

$$\Delta\rho(t) = -\frac{1}{i\hbar}\int_{-\infty}^{t} e^{-i(t-t')\hat{\mathcal{H}}/\hbar}[\hat{A}, \rho]e^{i(t-t')\hat{\mathcal{H}}/\hbar}F(t')dt'. \tag{27}$$

Then, ease of work can be obtained by introducing the $a^{\times}$ operator that works on other operators b with the following definition

$$a^{\times}b = [a, b], \tag{28}$$

which from the definition applies

$$e^{a^{\times}}b = e^{a}be^{-a} \tag{29}$$

Equations (26) and (27) can be written with this new notation to

$$\frac{d}{dt}\rho'(t) = \frac{1}{i\hbar}(\hat{\mathcal{H}}^{\times} + \hat{\mathcal{H}}'(t)^{\times})\rho'(t) \tag{30}$$

$$\Delta\rho(t) = -\frac{1}{i\hbar}\int_{-\infty}^{t} e^{-i(t-t')\hat{\mathcal{H}}^{\times}/\hbar}[\hat{A}, \rho]F(t')dt'. \tag{31}$$

This form will later show clearly the similarity between (21) and (27).

The response $\Delta\hat{B}(t)$ to the amount of $\hat{B}$ statistically is

$$\begin{aligned}\Delta\hat{B}(t) &= \mathrm{Tr}\,\Delta\rho(t)\hat{B} \\ &= -\frac{1}{i\hbar}\mathrm{Tr}\int_{-\infty}^{t} e^{-i(t-t')\hat{\mathcal{H}}/\hbar}[\hat{A}, \rho]\times \\ &\qquad e^{i(t-t')\hat{\mathcal{H}}/\hbar}\hat{B}F(t')dt' \\ &= -\frac{1}{i\hbar}\mathrm{Tr}\int_{-\infty}^{t}[\hat{A}, \rho]\hat{B}(t-t')F(t')dt'\end{aligned} \tag{32}$$

where $\hat{B}(t)$ is the Heisenberg representation for the $\hat{B}$ operator that satisfies the equation

$$\frac{d\hat{B}(t)}{dt} = \frac{1}{i\hbar}[\hat{B}(t), \hat{\mathcal{H}}], \qquad \hat{B}(0) = \hat{B}. \tag{33}$$

The equation (32) is a quantum mechanical version of (23).

So, the response function is

$$\phi_{BA}(t) = \frac{1}{i\hbar}\mathrm{Tr}[\hat{A}, \rho]\hat{B}(t) \tag{34}$$

or can be written (because of the trace invariance nature) as

$$\phi_{BA}(t) = \frac{1}{i\hbar}\mathrm{Tr}[\hat{A}, \rho]\hat{B}(t) \tag{35}$$

which is the quantum mechanical version of Equation (24). The **General susceptibility** (according to the equation (6)) is

$$\chi_{BA}(\omega) = \int_{0}^{\infty} e^{-i\omega t}\left(\frac{1}{i\hbar}\mathrm{Tr}[\rho, \hat{A}]\hat{B}(t)\right)dt. \tag{36}$$

2.5 Linear Responses in Quantum Theory

A response function for the macroscopic order associates the physical changes in the ensemble means of the physical observables $\langle B(t)\rangle$ with an external force $f(t)$. The application of linear response theory is limited to cases where $\langle B(t)\rangle$ changes linearly with force. Therefore assume that $f(t)$ is weak enough to ensure that the response is linear. Assume that the system is in thermal equilibrium before the external forces work.

In the thermal balanced state, the system is characterized by a density operator

$$\rho_0 = \frac{1}{Z}e^{-\beta\hat{\mathcal{H}}_0} \quad ; \quad Z = \mathrm{Tr}\ e^{-\beta\hat{\mathcal{H}}_0}, \tag{37}$$

where $\hat{\mathcal{H}}_0$ is Hamiltonan (effective), Z is a large canonical ensemble partition function, and β =

q/k_BT. The review is only in the linear part of the response, so it can be considered a form of $f(t)$ which results in a linear time-time disorder in the total Hamiltonian $\hat{\mathcal{H}}$ is:

$$\hat{\mathcal{H}}_1 = -\hat{A}f(t) \quad ; \quad \hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1, \tag{38}$$

where $\hat{A}$ is a fixed operator. As a result, the operator density $\rho(t)$ becomes a time-catcher, and so does the ensemble average of the operator $\hat{B}$:

$$\langle \hat{B}(t) \rangle = \mathrm{Tr}\{\rho(t)\hat{B}\}. \tag{39}$$

Linear response (as in Eq. (2)) is

$$\langle \hat{B}(t) \rangle - \langle \hat{B} \rangle = \int_{-\infty}^{t} dt' \; \phi_{BA}(t-t')f(t'), \tag{40}$$

with $\langle \hat{B} \rangle = \langle \hat{B}(t=-\infty) \rangle = \mathrm{Tr}\{\rho_0 \hat{B}\}$, $f(t)$ considered vanishing on $t \to \infty$. The nature of this equation is the same as the equation (2).

The general susceptibility (after undergoing the Fourier transform in the equation (40)) can be expressed as

$$\chi_{BA}(z) = \frac{1}{2\pi} \int_0^{\infty} \phi_{BA}(t) e^{izt} dt. \tag{41}$$

where $z = z_1 + iz_2$ is a complex variable. If $\int_0^{\infty} |\phi_{BA}| e^{-\epsilon t} dt$ is considered to be finite (*finite*) at the $\epsilon \to 0^+$ limit, the opposite relationship is

$$\phi_{BA}(t) = \int_{-\infty+i\epsilon}^{\infty+i\epsilon} \chi_{BA}(z) e^{-izt} dz; \qquad \epsilon > 0. \tag{42}$$

This equation shows that $\chi_{BA}(z)$ is an analytic function in the upper half of the complex plane z.

The changes of the system can be uniquely determined by $\rho_0 = \rho(-\infty)$ and $f(t)$, provided the external interference is given slowly or adiabatically. This can be done by replacing $f(t')$ in (40) with $f(t')e^{\epsilon t'}$, $\epsilon > 0$. This style disappears at $t' \to -\infty$, and additional unwanted effects can be removed by taking the $\epsilon \to 0^+$ limit. So, in terms of this 'general' Fourier transformation

$$\langle \hat{B}(\omega) \rangle = \lim_{\epsilon \to 0^+} \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\langle \hat{B}(t) \rangle - \langle \hat{B} \rangle \right) e^{i\omega t} e^{-\epsilon t} dt, \tag{43}$$

equation (40) transforms into

$$\langle \hat{B}(\omega) \rangle = \chi_{BA}(\omega) f(\omega), \tag{44}$$

where $\chi_{BA}(\omega)$ is the boundary condition for the analytic function $\chi_{BA}(z)$ on the real axis:

$$\chi_{BA}(\omega) = \lim_{\epsilon \to 0^+} \chi_{BA}(z = \omega + i\epsilon). \tag{45}$$

This is the general susceptibility[4].

2.6 Quantum Response Function

The equation for the response function in terms of operator $\hat{B}$ and $\hat{A}$, and unperturbed Hamiltonian $\hat{\mathcal{H}}_0$, is

$$\begin{aligned}\langle \hat{B}(t) \rangle - \langle \hat{B} \rangle &= \mathrm{Tr}\{(\rho(t) - \rho_0)\hat{B}\} \\ &= \frac{i}{\hbar} \mathrm{Tr} \left\{ \int_{-\infty}^{t} [\hat{A}_0(t'-t), \rho_0] \hat{B} f(t') \; dt' \right\}\end{aligned}$$

and by using the invariance on track experiencing cyclic permutation, then obtained the low-order terms

$$\begin{aligned}\langle \hat{B}(t) \rangle - \langle \hat{B} \rangle &= \frac{i}{\hbar} \int_{-\infty}^{t} \mathrm{Tr}\{\rho_0 [\hat{B}, \hat{A}_0(t'-t)]\} f(t') dt' \\ &= \frac{i}{\hbar} \int_{-\infty}^{t} \langle [\hat{B}_0(t), \hat{A}_0(t')] \rangle_0 f(t') dt'\end{aligned} \tag{46}$$

analogous with response function defition, which produce

$$\phi_{BA}(t-t') = \frac{i}{\hbar} \theta(t-t') \langle [\hat{B}(t), \hat{A}(t')] \rangle, \tag{47}$$

with a unit step function are introduced, namely for and for $t = 0$. This equation, which is expressed in microscopic quantity, called the Kubo formula for the response function[8].

2.7 Quantum Green Function

The Green function is defined as:

$$\begin{aligned}G_{BA}(t-t') &\equiv \langle\langle \hat{B}(t); \hat{A}(t') \rangle\rangle \\ &\equiv -\frac{i}{\hbar} \theta(t-t') \langle [\hat{B}(t), \hat{A}(t')] \rangle \\ &= -\phi_{BA}(t-t').\end{aligned} \tag{48}$$

This Green function is also called **double-time Green function** or **retarded Green function**, which is the previous response function, but with the opposite sign. Laplace transform of the function using (41), is

$$\begin{aligned}G_{BA}(\omega) \equiv \langle\langle \hat{B}; \hat{A} \rangle\rangle_\omega &= \lim_{\epsilon \to 0^+} G_{BA}(z = \omega + i\epsilon) \\ &= \lim_{\epsilon \to 0^+} G_{BA}(t) e^{i(\omega + i\epsilon)t} dt = -\chi_{BA}(\omega).\end{aligned} \tag{49}$$

If $\hat{A}$ and $\hat{B}$ are dimensionless operators, then $G_{BA}(\omega)$ or $\chi_{BA}(\omega)$ has the inverse dimension of energy .

At $t' = 0$, the derivative of the Green function above to t is

$$\frac{d}{dt}G_{BA}(t) = -\frac{i}{\hbar}\Big(\delta(t)\langle[\hat{B}(t),\hat{A}]\rangle + \theta(t)\langle[d\hat{B}(t)/s,\hat{A}]\rangle\Big) = -\frac{i}{\hbar}\Big(\delta(t)\langle[\hat{B},\hat{A}]\rangle - \frac{i}{\hbar}\theta(t)\langle[[\hat{B}(t),\hat{\mathcal{H}}],\hat{A}]\rangle\Big). \quad (50)$$

The Fourier transform of the above equation into the Green function motion equation:

$$\hbar\omega\langle\langle\hat{B};\hat{A}\rangle\rangle_\omega - \langle\langle[\hat{B},\hat{\mathcal{H}}];\hat{A}\rangle\rangle_\omega = \langle[\hat{B},\hat{A}]\rangle. \quad (51)$$

The ω labels indicate the Fourier transform (49), and $\hbar\omega$ is short for $\hbar(\omega + i\epsilon)$ with $\epsilon \to 0^+$. In many applications, $\hat{A}$ and $\hat{B}$ are the same Hermitian operator, so the right side of the equation (51) above disappears, and the equation can be derived a second time. Because $\langle[[[\hat{A}(t),\hat{\mathcal{H}}],\hat{\mathcal{H}}],\hat{A}]\rangle$ equals $-\langle[[\hat{A}(t),\hat{\mathcal{H}}],\hat{A}]\rangle$, Green equation of motion for $\langle\langle[\hat{A},\hat{\mathcal{H}}];\hat{A}\rangle\rangle_\omega$ to be

$$(\hbar\omega)^2\langle\langle\hat{A};\hat{A}\rangle\rangle_\omega + \langle\langle[\hat{A},\hat{\mathcal{H}}];[\hat{A},\hat{\mathcal{H}}]\rangle\rangle_\omega = \langle[[\hat{A},\hat{\mathcal{H}}],\hat{A}]\rangle. \quad (52)$$

The pair of equations (51) and (52) will underlie the applied linear response theory.

In accordance with the understanding about the response function $K_{BA}(t)$,

$$K_{BA}(t) = \frac{i}{\hbar}\langle[\hat{B}(t),\hat{A}]\rangle = \frac{i}{\hbar}\langle[\hat{B},\hat{A}(-t)]\rangle \quad (53)$$

and $K_{BA}(z) = 2i\chi''_{BA}(z)$ we get

$$K_{BA}(\omega) = 2i\chi''_{BA}(\omega) = -2iG''_{BA}(\omega). \quad (54)$$

This equation can be written down

$$2i\int_{-\infty}^{\infty}\chi''BA(\omega)e^{-i\omega t}d\omega = \frac{i}{\hbar}\langle[\hat{B}(t),\hat{A}]\rangle \quad (55)$$

and by selecting $t = 0$, we will get the following **addition rule**

$$2\hbar\int_{-\infty}^{\infty}\chi''_{BA}(\omega)d\omega = \langle[\hat{B},\hat{A}]\rangle, \quad (56)$$

The Green function in (51) must meet these summation rules, and the thermal equalization of (51) and (56) must be the same. The equation (56) is only the first term of this whole series of addition rules.

The n-time derivative of $\hat{B}(t)$ can be written as

$$\frac{d^n}{dt^n}\hat{B}(t) = \left(\frac{i}{\hbar}\right)^n\mathcal{L}^n\hat{B}(t) \quad \text{with} \quad \mathcal{L}\hat{B}(t) \equiv [\hat{\mathcal{H}},\hat{B}(t)]. \quad (57)$$

A n derivative on both sides of the equation (55) results

$$\frac{i}{\pi}\int_{-\infty}^{\infty}(-i\omega)^n\chi''_{BA}(\omega)e^{-i\omega t}d\omega = \left(\frac{i}{\hbar}\right)^{n+1}\langle[\mathcal{L}^n\hat{B}(t),\hat{A}]\rangle. \quad (58)$$

Then the normalized "spectral weight function" was introduced

$$F_{BA}(\omega) = \frac{1}{\chi'_{BA}(0)}\frac{1}{\pi}\frac{\chi''_{BA}(\omega)}{\omega}, \quad \int_{-\infty}^{\infty}F_{BA}(\omega)d\omega = 1. \quad (59)$$

The $F_{BA}(\omega)$ normalization is the result of the Kramers-Kronig relationship. The n-order of ω of the $F_{BA}(\omega)$ spectral weight function, is defined

$$\langle\omega^n\rangle_{BA} = \int_{-\infty}^{\infty}\omega^n F_{BA}(\omega)d\omega. \quad (60)$$

which permits a link between the n-derivative at $t = 0$ is written

$$\chi'_{BA}(\omega)\langle(\hbar\omega)^{n+1}\rangle_{BA} = (-1)^n\langle[\mathcal{L}^n\hat{B},\hat{A}]\rangle. \quad (61)$$

This is an addition rule that associates spectral frequencies with thermal expectation values of operators obtained from $\hat{B}$, $\hat{A}$, and $\hat{\mathcal{H}}$. If $\hat{B} = \hat{A} = \hat{A}^\dagger$, then $F_{BA}(\omega)$ is an even function in ω, and all the odd parts are gone. In this case, the even part is

$$\chi'_{AA}(0)\langle(\hbar\omega)^{2n}\rangle_{AA} = -\langle[\mathcal{L}^{2n-1}\hat{A},\hat{A}]\rangle. \quad (62)$$

2.8 Relaxation Function

This section will discuss the behavior of the $\phi_{BA}(t)$ response function in the $t \to \infty$ approach. It can go to a certain value or it can not. If it goes to a certain value, then according to Kubo (1957)[6], it must fulfill

$$\lim_{t\to\infty}\phi_{BA}(t) = 0 \quad \text{(if there is a limit).} \quad (63)$$

If the application of continuous interruption (f is constant) from $t = -\infty$ to $t = 0$ is stopped at $t = 0$, then the ΔB response will stretch according to the following equation

$$\Delta B(t) = \int_{-\infty}^{0} \phi_{BA}(t-t')f dt' \\ = f\int_{t}^{\infty} \phi_{BA}(t')dt', \qquad t>0. \tag{64}$$

So functions can be defined

$$\varPhi_{BA}(t) = \lim_{\epsilon\to 0+}\int_{t}^{\infty} \phi_{BA}(t')e^{-\epsilon t'}dt' \tag{65}$$

which is called **relaxation function**. This function explains the relaxation of ΔB after the outside has been removed.

Theorem and form of this relaxation function, according to Kubo (1957)[6] is: The $\chi_{BA}(\omega)$ acceptability can be calculated using the following formula:

$$\chi_{BA}(\omega) = \lim_{\epsilon\to 0+}\int_{min}^{max} \phi_{BA}(t)e^{-i\omega t-\epsilon t}d \\ = \lim_{\epsilon\to 0+}\frac{1}{i\omega+\epsilon}\left\{\dot{\phi}_{BA}(0) + \int_{0}^{\infty}\phi_{BA}(t)e^{-i\omega t-\epsilon t}dt\right\} \\ = \varPhi_{BA}(0) - i\omega\int_{0}^{\infty}\varPhi_{BA}(t)e^{-i\omega t}dt. \tag{66}$$

The form of the relaxation function is:

$$\varPhi_{BA}(t) = \frac{i}{\hbar}\int_{t}^{\infty}\mathrm{Tr}\{\rho[B(t'),A]\}dt' \tag{67a}$$

$$= \int_{0}^{\beta}\mathrm{Tr}\{\rho A(-i\hbar\lambda)B(t)d\lambda\} - \beta\,\mathrm{Tr}\{\rho A^0B^0\} \tag{67b}$$

3 Application of Linear Response Theory

3.1 Magnetic Susceptibility

A uniform magnetic field of $\mathbf{H}(t)$ is applied to a magnetic system, the total magnetic moment is written as $\mathbf{M}$. The energy perturbation caused by $\mathbf{H}(t)$ is

$$\hat{\mathcal{H}}'(t) = -\mathbf{M}\mathbf{H}(t). \tag{68}$$

The natural motion of the magnetic moment when there is no external field is represented by $\mathbf{M}(t)$. Then, the $\phi_{\mu\nu}(t)$ response function for magnetization in the μ-direction when the external field $\mathbf{H}(t)$ points in the ν-direction, according to (46) is

$$\phi_{\mu\nu}(t) = \frac{i}{\hbar}\langle[M_\mu(t), M_\nu]\rangle \\ = \int_{0}^{\beta}\langle\dot{M}_\nu(-i\hbar\lambda)M_\mu(t)\rangle d\lambda, \tag{69}$$

and according to (67), the relaxation function is

$$\varPhi_{\mu\nu}(t) = \int_{0}^{\beta}\langle(M_\mu(-i\hbar\lambda) - M_\mu^0)(M_\mu(t) - M_\mu^0)\rangle d\lambda \tag{70}$$

where M_ν^0 and M_μ^0 are the diagonal portion of M_ν and M_μ of unperturbed Hamiltonan $\hat{\mathcal{H}}$.

If a system is measured in one volume, this admittance will become a magnetic susceptibility. So the magnetic susceptibility tensor can be expressed in either $\phi_{\mu\nu}(t)$ or $\Phi_{\mu\nu}(t)$. The simplest equation is to use the equation (66)

$$\chi_{\mu\nu}(\omega) = \varPhi_{\mu\nu}(0) - i\omega\int_{0}^{\infty}\varPhi_{\mu\nu}(t)e^{-i\omega t}dt. \tag{71}$$

The static susceptibility is obtained in the form

$$\chi_{\mu\nu}(0) = \int_{0}^{\beta}\langle(M_\nu(-i\hbar\lambda) - M_\nu^0)(M_\mu - M_\mu^0)\rangle d\lambda \tag{72}$$

which is a susceptibility for an isolated system, and does not have to be the same as an isothermal susceptibility in the form

$$\chi_{\mu\nu}^T = \int_{0}^{\beta}\langle(M_\nu(-i\hbar\lambda)M_\mu)\rangle d\lambda - \beta\langle M_\nu\rangle\langle M_\mu\rangle. \tag{73}$$

In the equation $\chi_{\mu\nu}(0)$, the diagonal portion of M_ν and M_μ is subtracted. This is related to the fact that magnetization in an isolated system takes place adiabatically, the chance of occupation from its energy levels remains unchanged. But in the isothermal process these levels change to $\chi_{\mu\nu}^T$. This difference will diminish if the environmental role is taken into account on a larger scale.

In the classic approach, Equation (72) becomes

$$\chi_{\mu\nu}(0) = \frac{1}{kT}\langle(M_\nu - M_\nu^0)(M_\mu - M_\mu^0)\rangle. \tag{74}$$

meanwhile Equation (73) becomes

$$\chi_{\mu\nu}^T = \frac{1}{kT}\langle(M_\nu\langle M_\nu\rangle)(M_\mu - \langle M_\mu\rangle)\rangle. \tag{75}$$

The last equation was first revealed by Kirkwood (Kirkwood, 1939)[9] for the classical theory of dielectric polarization. Equation (74) is its extension to adiabatic susceptibility, and Equation (71) for complex susceptibility which is common for unequilibrium states. This is proof that similar equations are also obtained for dielectric polarization.

3.2 Electric Conductivity Tensor

As an applied example, the link between the electrical conductivity tensor and the Green function will be reviewed. Assume that the electric field $E(t)$ is given adiabatically, which is uniform in space and changes periodically with a frequency of ω

$$\mathbf{E}(t) = \mathbf{E}\cos\omega t.$$

The perturbation operator is

$$\hat{H}_t^1 = -\sum e_j(\hat{\mathbf{E}}\mathbf{x}_j)\cos\omega t\ e^{\epsilon t} \tag{76}$$

(where e_j is the jth particle load, and the sum includes all particles with coordinates $\mathbf{x}_j$). The perturbation (76) causes an electric current to appear in the system

$$\hat{j}_\mu(t) = \int_{-\infty}^{\infty} \langle\langle j_\mu(t);\ H_\tau^1(\tau)\rangle\rangle d\tau, \tag{77}$$

with

$$\begin{aligned} \hat{H}_\tau^1(\tau) &= \hat{H}^1(\tau)\cos\omega\tau\ e^{\epsilon\tau}, \\ \hat{H}^1(t) &= -\sum_{j,\mu} e_j E_\mu \dot{\hat{x}}_{\mu\mathrm{J}}(\tau), \\ \hat{j}_\mu(t) &= \sum_j e_j \dot{\hat{x}}_{\mu\mathrm{J}}(t), \end{aligned} \tag{78}$$

$\hat{j}_\mu$ is the current density operator, if the system volume is considered one unit volume. Integration of each part makes the equation (77) writeable in form

$$j_\mu(t) = -\mathrm{Re}\left\{ \int_{-\infty}^{\infty} \langle\langle j_\mu;\ \dot{\hat{H}}^1(\tau)\rangle\rangle \frac{e^{i\omega t+\epsilon\tau}d\tau}{i\omega+\epsilon} + \langle[j_\mu(0), H^1(0)]\rangle e^{i\omega t+\epsilon t}\frac{1}{\omega - i\epsilon} \right\}. \tag{79}$$

It should be noted that

$$\begin{aligned} \dot{\hat{H}}^1(\tau) &= -(\hat{E}\mathbf{j}(\tau)),\ \text{and} \\ [\dot{\hat{x}}_{\mu\mathrm{J}_1},\ \hat{x}_{\nu j_2}] &= -\frac{i}{m}\delta_{\mu\nu}\delta_{j_1 j_2} \qquad (\hbar = 1) \end{aligned} \tag{80}$$

so from this equation is obtained

$$j_\mu(t) = \mathrm{Re}\{\sigma_{\mu\nu}(\omega)E_\nu e^{i\omega t+\epsilon t}\}, \tag{81}$$

$$\sigma_{\mu\nu}(\omega) = -\frac{ie^2 n}{m\omega}\delta_{\mu\nu} + \int_{-\infty}^{\infty} \langle\langle j_\mu(0); j_\nu(\tau)\rangle\rangle \frac{e^{i\omega t+\epsilon\tau}}{i\omega+\epsilon} d\tau \tag{82}$$

where $\sigma_{\mu\nu}(\omega)$ is the tensor of electrical conductivity, and n is the number of electrons per unit volume. The first term in (82) is related to the electrical conductivity of a system that has a free charge and does not interact with other particles. As $\omega \to \infty$ increases the second term decreases faster than the first term ($\lim_{\omega\to\infty} \mathrm{Im}\ \omega\sigma_{\mu\nu}(\omega) = -e^2 n\delta_{\mu\nu}/m$), and the system behaves as a collection of free loads. Equation (82) can be written in a different but equivalent form, and integrating it byτ, it becomes

$$\sigma_{\mu\nu}(\omega) = -\frac{ie^2 n}{m\omega}\delta_{\mu\nu} + \frac{e^{-\frac{\omega}{\beta}} - 1}{\omega} \int_{-\infty}^{\infty} \langle j_\mu(0) j_\nu(t)\rangle e^{-i\omega t} dt. \tag{83}$$

This equation is known as Nyquist theorem, which is then generalized by Callen and Welton for the case of quantum mechanics[10]; this equation connects electrical conductivity with changes in current.

3.3 Linear Responses to Heisenberg Ferromagnet

In this section, we will review the linear response function which is applied to the Heisenberg ferromagnet case in three dimensions, with Hamiltonian

$$\hat{\mathcal{H}} = -\frac{1}{2}\sum_{i\neq j}\mathcal{J}(ij)\mathbf{S}_i\cdot\mathbf{S}_j, \tag{84}$$

where $\mathbf{S}_i$ is the spin on the i ion, which is placed on a Bravais lattice at the position of $\mathbf{R}_i$. The Fourier transform of the exchange coupling, provided $\mathcal{J}(ii) \equiv 0$, is

$$\mathcal{J}(q) = \frac{1}{N}\sum_{ij} e^{-i\mathbf{q}\cdot(\mathbf{R}_i - \mathbf{R}_j)} = \sum_j \mathcal{J}(ij)e^{-i\mathbf{q}\cdot(\mathbf{R}_i - \mathbf{R}_j)}, \tag{85}$$

and vice versa

$$\mathcal{J}(ij) = \frac{1}{N}\sum_{\mathbf{q}} \mathcal{J}(\mathbf{q}) e^{i\mathbf{q}\cdot(\mathbf{R}_i \mathbf{R}_j)} = \frac{V}{N(2\pi)^3}\int \mathcal{J}(\mathbf{q}) e^{i\mathbf{q}\cdot(\mathbf{R}_i - \mathbf{R}_j)} d\mathbf{q}, \quad (86)$$

relies on the review of $\mathbf{q}$ (which is defined in the primitive Brillouin zone) as a discrete or continuous variable, and will be considered as a discrete variable. N is the number of spins, V is the volume, and the reversal of symmetry from the Bravais lattice causes that $\mathcal{J}(\mathbf{q}) = \mathcal{J}(-\mathbf{q}) = \mathcal{J}^*(\mathbf{q})$. The maximum value of $\mathcal{J}(\mathbf{q})$ is considered to be $\mathcal{J}(\mathbf{q} = \mathbf{0})$, which in that state the equilibrium state occurs at zero temperature, or in other words at the ground state, i.e. the ferromagnet:

$$\langle \mathbf{S}_i \rangle = S\hat{\mathbf{z}} \quad \text{on} \quad T = 0, \quad (87)$$

where $\hat{\mathbf{z}}$ is the unit vector on the z-axis, which is constructed as the direction of magnetization caused by an infinitesimal magnetic field. This result is appropriate, but as the temperature rises above zero, a number of approaches are needed. As a first step, the $\langle \mathbf{S}_i \rangle = \langle \mathbf{S} \rangle$ thermal expectation value can be written (after the rearrangement of the terms)

$$\hat{\mathcal{H}} = \sum_i \hat{\mathcal{H}}_i - \frac{1}{2}\sum_{i\neq j} \mathcal{J}(ij)(\mathbf{S}_i - \langle \mathbf{S}\rangle)\cdot(\mathbf{S}_j - \langle \mathbf{S}\rangle), \quad (88a)$$

with

$$\hat{\mathcal{H}}_i = -S_i^z \mathcal{J}(\mathbf{0})\langle S^z\rangle + \frac{1}{2}\mathcal{J}(\mathbf{0})\langle S^z\rangle^2, \quad (88b)$$

and $\langle \mathbf{S}\rangle = \langle S^z\rangle \hat{\mathbf{z}}$. The second term is ignored based on the average field approach, which results in the original multi-fold Hamiltonian being the sum of the N Hamiltonian free single spin (88b). In this approach, $\langle S^z\rangle$ is determined by *self-consistence equations* is

$$\langle S^z\rangle = \sum_{M=-S}^{+S} M\ e^{\beta M \mathcal{J}(\mathbf{0})\langle S^z\rangle} / \sum_{M=-S}^{+S} e^{\beta M \mathcal{J}(\mathbf{0})\langle S^z\rangle} \quad (89a)$$

(the last term in (88b) does not affect thermal averages). The equation (89a) in the low temperature approach is

$$\langle S^z\rangle \simeq S - e^{-\beta S \mathcal{J}(\mathbf{0})}. \quad (89b)$$

Review the Green function, to include initial order effects on two-place correlations, with

$$G^{\pm}(ii', t) = \langle\langle S_i^+(t); S_{i'}^-\rangle\rangle. \quad (90)$$

According to (51), the change in time according to $G^{\pm}(ii', t)$ depends on the operator

$$[S_i^+, \hat{\mathcal{H}}] = -\frac{1}{2}\sum_j \mathcal{J}(ij)(-2S_i^+ S_j^z + 2S_i^z S_j^+). \quad (91)$$

The inclusion of this commutator in the equation (51) leads to the link between the original Green function and the new Green function more clearly. Through this equation of motion the new function can be stated with further new functions, thus forming infinite coupled equations. The answer to the approach can be obtained by using the condition that the expected value of S_i^z is close to the saturation value when it is at a low temperature. So in this approach, S_i^z has to be almost timeless, in other words $S_i^z \simeq \langle S^z\rangle$. In 'this phase-random approach' (*random phase approximation* (RPA)) the commutator shrank to

$$[S_i^+, \hat{\mathcal{H}}] \simeq -\sum_j \mathcal{J}(ij)\langle S^z\rangle (S_j^+ - S_i^+) \quad (92)$$

and the equation of motion leads to the following set of linear equations:

$$\hbar\omega G^{\pm}(ii', \omega) + \sum_j \mathcal{J}(ij)\langle S^z\rangle\{G^{\pm}(ji', \omega) - G^{\pm}(ii', \omega)\} = \langle [S_i^+, S_{i'}^-]\rangle = 2\langle S^z\rangle \delta_{ii'}. \quad (93)$$

4 Conclusion

4.1 Linear Response Theory

1. The response $R(t)$ given by an order because the external force $f(t')$ given to him at time t' can be described by the response function $\phi(t - t')$ with the equation :

$$\mathcal{R}(t) = \int_{-infty}^{t} \phi(t - t') f(t') dt' \quad (94)$$

assuming the style and response meet the **rule of causality** and the assumption of **linearity** the response to its style.

2. The link between **general susceptibility** and the response function is

$$\chi(\omega) = \int_0^{\infty} \phi(\tau) e^{i\omega\tau} d\tau \quad (95)$$

3. The properties possessed by general susceptibility are:
   (a) The real and imaginary parts have properties

$$\mathrm{Re}\chi(-\omega) = \mathrm{Re}\chi(\omega), \text{and} \quad (96)$$

$$\mathrm{Im}\chi(-\omega) = -\mathrm{Im}\chi(\omega) \quad (97)$$

(b) $\chi(\omega)$ is analytic in the upper half of the complex plane ω, which is caused by the rule of causality.

(c) **Dispersion relation / Kramers-Kronig relation** connecting real and imaginary parts to general susceptibility is

$$\mathrm{Re}\chi(\omega) = \frac{2P}{\pi}\int_0^\infty \frac{d\omega'\ \omega'\ \mathrm{Im}\chi(\omega')}{\omega'^2-\omega^2} \quad (98)$$

and

$$\mathrm{Im}\chi(\omega) = -\frac{2P\omega}{\pi}\int_0^\infty \frac{d\omega'\ \mathrm{Re}\chi(\omega')}{\omega'^2-\omega^2}. \quad (99)$$

4.2 Linear Responses in Quantum Theory

1 The form of the link between responses and disturbances in quantum theory is

$$\begin{aligned}\langle\hat{B}(t)\rangle - \langle\hat{B}\rangle &= \frac{i}{\hbar}\int_{-\infty}^{t} \mathrm{Tr}\{\rho_0[\hat{B},\hat{A}_0(t'-t)]\}\times f(t')dt' \\ &= \frac{i}{\hbar}\int_{-\infty}^{t}\langle[\hat{B}_0(t),\hat{A}_0(t')]\rangle_0 f(t')dt'\end{aligned} \quad (100)$$

2 The form of the quantum response function is

$$\phi_{BA}(t-t') = \frac{i}{\hbar}\theta(t-t')\langle[\hat{B}(t),\hat{A}(t')]\rangle, \quad (101)$$

3 The form of the fluctuation-dissipation theorem in this theory viz

$$S_{BA}(\omega) = 2\hbar\frac{1}{1-e^{\beta\hbar\omega}}\chi''_{BA}(\omega), \quad (102)$$

4 The link between the snooze function and the response function is

$$\begin{aligned}G_{BA}(t-t') &\equiv \langle\langle\hat{B}(t);\hat{A}(t')\rangle\rangle \\ &\equiv -\frac{i}{\hbar}\theta(t-t')\langle[\hat{B}(t),\hat{A}(t')]\rangle \\ &= -\phi_{BA}(t-t').\end{aligned} \quad (103)$$

5 The link between the Green delay function and general susceptibility is

$$\begin{aligned}G_{BA}(\omega) &\equiv \langle\langle\hat{B};\hat{A}\rangle\rangle_\omega = \lim_{\epsilon\to 0^+} G_{BA}(z=\omega+i\epsilon) \\ &= \lim_{\epsilon\to 0^+} G_{BA}(t)e^{i(\omega+i\epsilon)t}dt = -\chi_{BA}(\omega).\end{aligned} \quad (104)$$

4.3 Application of Response Function in Some Elementary Processes

1 Magnetic susceptibility to systems with a total magnetic moment of $\mathbf{M}$ that has external interference $\mathbf{H}(t)$ is:

(a) The static susceptibility is:

$$\chi_{\mu\nu}(0) = \int_0^\beta \langle(M_\nu(-i\hbar\lambda) - M_\nu^0)(M_\mu - M_\mu^0)\rangle d\lambda \quad (105)$$

(b) The isothermal susceptibility is

$$\chi_{\mu\nu}^T = \int_0^\beta \langle(M_\nu(-i\hbar\lambda)M_\mu)\rangle d\lambda - \beta\langle M_\nu\rangle\langle M_\mu\rangle. \quad (106)$$

2 The form of the electrical conductor tensor that is experiencing outside interference in the form of an electric field $\mathbf{E}(t) = \mathbf{E}\cos\omega t$. is

$$\sigma_{\mu\nu}(\omega) = -\frac{ie^2n}{m\omega}\delta_{\mu\nu} + \int_{-\infty}^{\infty}\langle\langle j_\mu(0); j_\nu(\tau)\rangle\rangle\frac{e^{i\omega t+\epsilon\tau}}{i\omega+\epsilon}d\tau \quad (107)$$

or it can also be written in a different form, known as the Nyquist theorem, viz

$$\sigma_{\mu\nu}(\omega) = -\frac{ie^2n}{m\omega}\delta_{\mu\nu} + \frac{e^{-\frac{\omega}{\beta}}-1}{\omega}\int_{-\infty}^{\infty}\langle j_\mu(0)j_\nu(t)\rangle e^{-i\omega t}dt. \quad (108)$$

3 Linear response to Heisenberg Ferromagnet, with the general form of susceptibility is

$$\chi_{xx}(\mathbf{q},\omega) = \chi_{yy}(\mathbf{q},\omega) = \frac{1}{4}\{\chi_{+-}(\mathbf{q},\omega) + \chi_{-+}(\mathbf{q},\omega)\}. \quad (109)$$

with

$$\chi_{+-}(\mathbf{q},\omega) = \frac{2\langle S^z\rangle}{E_{\mathbf{q}}-\hbar\omega} + i\pi 2\langle S^z\rangle\delta(\hbar\omega - E_{\mathbf{q}}). \quad (110a)$$

and

$$\chi_{-+}(\mathbf{q},\omega) = \frac{2\langle S^z\rangle}{E_{\mathbf{q}}+\hbar\omega} - i\pi 2\langle S^z\rangle\delta(\hbar\omega + E_{\mathbf{q}}), \quad (110b)$$

4.4 Future Study

The discussion that has been carried out in this article is limited to a linear process. Further research can be developed to examine non-linear response functions, the more general response functions. This non-linear response function is expected to have a wider scope of coverage and a better level of accuracy than the response function, which is limited to linear areas only.

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