Mahalanobis Fuzzy C-Means Clustering with Spatial Information for Image Segmentation

Abstrak

Algoritma segmentasi Fuzzy C-Means dapat diimplementasikan pada segmentasi citra berdasarkan jarak mahalanobis; Namun, metode ini hanya perlu mempertimbangkan situasi ruang warna, bukan sistem ketetanggaan citra. itu adalah efek proses deteksi tepi yang tidak berjalan dengan baik dan menghasilkan akurasi yang kurang dalam hasil segmentasi. Pada artikel ini, kami mengusulkan metode baru untuk segmentasi citra dengan Mahalanobis fuzzy C-means Spatial information (MFCMS). Metode yang diusulkan menggabungkan ruang fitur dan citra informasi lingkungan (informasi spasial) untuk meningkatkan akurasi hasil segmentasi pada citra. MFCMS terdiri dari dua Langkah, modul histogram threshold untuk langkah pertama dan modul MFCMS untuk langkah kedua. Modul Threshold Histogram digunakan untuk mendapatkan kondisi inisialisasi MFCMS untuk centroid cluster dan jumlah centroid. Hasil pengujian menunjukkan bahwa metode ini memberikan kinerja segmentasi yang lebih baik daripada kesalahan klasifikasi (ME) dan kesalahan area latar depan relatif (RAE) masing-masing sebesar 1,61 dan 3,48.

Kata kunci*: Histogram thresholding module, Segmentasi Citra, Mahalonbis Fuzzy C-Means Spatial Information*

Abstract

A fuzzy C-Means segmentation algorithm can be implemented in an image segmentation based on the Mahalanobis distance; However, this method only needs to consider the color space situation, not the neighborhood system of the image. It was an effective edge detection process unwell performed and generated less accuracy in segmentation results. In this article, we propose a new method for image segmentation with Mahalanobis fuzzy C-means Spatial information (MFCMS). The proposed method combines feature space and images of the information of the neighborhood (spatial information) to improve the accuracy of the result of segmentation on the image. The MFCMS consists of two steps, the histogram threshold module for the first step and the MFCMS module for the second step. The Histogram Threshold module is used to get the MFCMS initialization conditions for the cluster centroid and the number of centroids. Test results show that this method provides better segmentation performance than classification errors (ME) and relative foreground area errors (RAE) of 1.61 and 3.48, respectively.

Keywords: *Histogram thresholding module, Image Segmentation, Mahalonbis Fuzzy C-Means Spatial Information*

1. INTRODUCTION

In computer vision, image segmentation is the primary role of pattern recognition and image analysis, especially in color images [1]. Segmentation aims to divide a picture into a series of interconnected areas with similar properties like color, texture, and intensity. The resulting image segmentation is helpful in object recognition, facial recognition systems, biomedical image processing, and other computer vision applications. Several methods can be done to implement segmentation. Based on basic mathematics, some of the existing methods can be grouped into three categories: thresholding methods [2], statistics-based algorithms [3], and neural network-based clustering methods [4]. Although those segmentation strategies are extensively used, each has its benefits and limitations.

Feature space clustering is another method of segmentation approach. The fuzzy C-means (FCM) clustering algorithm [5] is more effective with considerable benefits. Unlike the complex clustering method, the FCM algorithm allows pixels to be associated with several clusters with different degrees of membership by assigning pixels to only one cluster. Therefore, the FCM algorithm makes more sense in real-world applications but is very sensitive to noise because pixel spatial information is ignored in the standard FCM algorithm.

Many modified FCM algorithms suggest using spatial information, especially pixel adjacency information, which results in more uniform segmentation. Ahmed [6] published an algorithm FCM-with-Constraint (FCM-S) that calculates the Euclidean distance of a pixel to the center of the cluster and the Euclidean distance of the neighboring pixel to the center of the cluster. In addition, the Euclidean distance from adjacent pixels to the cluster's center had to be calculated iteratively during the algorithm iteration, and that causes the algorithm time-consuming. Szilagyi [7] proposed an improved FCM (EnFCM) to speed up the clustering process based on the grayscale histograms. However, the smoothed picture might also lower the noise impact, and the element records are misplaced during the smoothing procedure, which may result in misclassifications.

The traditional Euclidean distance-based clustering method can be used for image segmentation. Euclidean distance describes the linear distance between two data points in feature space. It is susceptible to noise because it does not pay attention to data covariance as a dissimilarity measure. It will be ineffective if applied to the image whose pixel form clusters hyperelliptic in the feature space. On the other hand, when the Euclidean distance is applied in the clusters hyperelliptic, the Euclidean distance only takes the mean of cluster information. On the other side, Mahalanobis distance-based clustering method is more effective for clustering image pixels forming groups hyperelliptic the feature space. Therefore, Zhao [8] proposed FCM-based Mahalanobis Distance (FCM), and the addition of new rules on the objective function reflects the covariance of the cluster. However, this method only needs to take into account the position in the color space and does not take into account the image environment system. An effect of edge detection processes has been well performed and generated less accuracy in segmentation results.

In this article, we suggest methods for image segmentation with Mahalanobis fuzzy C-means Spatial information (MFCMS). The proposed method combines feature space and images with neighborhood or spatial information to improve the accuracy of image segmentation. The MFCMS consists of two steps, the histogram thresholding module, and the MFCMS module. The Histogram Threshold module is used to get the MFCMS initialization conditions of the cluster centroid and the number of centroids.

2. METHODS

2.1 Algorithm

Choosing the suitable color component to perform segmentation is a problem for segmentation results. The most used color space is RGB, but the same distance in the RGB space can mean no difference between the colors seen by the human eye and the components of the HSI. Ito [9] determined that the RGB components are highly correlated and the HSI components are highly independent. For that reason, color spaces HSI is applied to determine the histogram peaks. However, there are only two components that are implemented in this current research, i.e., hue H that carries the utmost information of image [10] and intensity I [11]. From this step, there are four sub-steps: get a histogram of the HSI color spaces H, I, determination of the peak, the formation of the cluster area, and the last, merging clusters. RGB image as input and convert to HSI. First, an extraction process histogram. The histogram obtained will be used as input in the next stage based on an equation (1-2).

$$h(i) = x_i \,, \quad (1)$$
$$h(i) = x_i \,, \quad (2)$$

Where $0 < i \leq L - 1$.

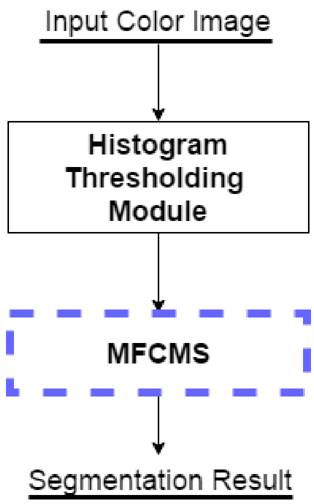

Fig 1. The Proposed Method

Second, after getting the histogram for H and I, the following step is to find the peak point for each histogram. The steps to find the peak, namely: step (1), form a new histogram based on the histogram obtained from the original image using $T_s(i)$ the following equation (3).

$$\boldsymbol{T_s(i)} = \frac{(s(i-5)+s(i-4)+\cdots+s(i)+\cdots+s(i+4)+s(i+5))}{11}, \quad (3)$$

Step (2), with s is the histogram H or I and met at $5 < i \leq L - 6$. Identification of the entire peak P_s of each histogram using equation (4) and Identification of all valleys V_s For each histogram, use equation (5).

$$P_s = \big((i, T_s(i))\,|\, T_s(i) > T_s(i-1) \text{ and } T_s(i) > T_s(i+1)\big), \quad (4)$$

$$V_s = \big((i, T_s(i))\,|\, T_s(i) < T_s(i-1) \text{ and } T_s(i) < T_s(i+1)\big), \quad (5)$$

Step (3) removes all the valleys and peaks based on the fuzzy rule base shown in equation (6).

$$IF(i \text{ is peak})\ AND\ \big(T_s(i+1) > T_s(i-1)\big) \quad (6)$$
$$THEN\ \big(T_s(i) = T\_s(i+1)\big)$$
$$IF(i \text{ is peak})\ AND\ \big(T_s(i+1) < T\_s(i-1)\big)$$
$$THEN\ \big(T_s(i) = T_s(i-1)\big)$$

$IF(i \text{ is valley})\ AND\ \left(T_s\,(i+1) > T\_s(i-1)\right)$
$THEN\ \left(T_s\,(i) = T\_s(i-1)\right)$
$IF(i \text{ is valley})\ AND\ \left(T_s\,(i+1) < T\_s(i-1)\right)$
$THEN\ \left(T_s\,(i) = T\_s(i+1)\right)$

Third, the Identification of all peaks in each histogram by the number of pixels greater than t_1.where t_1 is a minimum threshold number of pixels on peaks. The following initialization step to get a cluster area is as follows:

1. Get all possible initial c_i Centroid, the numeral of initial centroids, is obtained from the numeral of heights in the histogram H and I, which were identified by P_s
2. Identify each pixel X_i to the point nearest centroid c_i
3. elimination centroid c_i If the number of pixels is smaller than the Threshold t_2
4. re-classify each pixel to the nearest centroid c_i

Fourth is merging clusters into smaller ones formed from the previous process [12]. The steps for merging clusters are: step (1) set the number of clusters *c* and step (2) calculate the distance between clusters using r_{ij} as shown in equation (7).

$$r_{ij} = k_1 \beta^2, \tag{7}$$

Where β is obtained using equation (8) and k_1 and k_2 are obtained using equation (9) and equation (10).

$$\beta = \frac{\sqrt{k_1 k_2 - k_2}}{k_1 - k_2}, \tag{8}$$

$$k_1 = [\mu_i^T \textstyle\sum_j^{-1} \mu_i + \mu_j^T \textstyle\sum_j^{-1} \mu_j - 2\mu_i^T \textstyle\sum_j^{-1} \mu_j], \tag{9}$$

$$k_2 = [\,\mu_i^T \textstyle\sum_j^{-1} \mu_i + \mu_j^T \textstyle\sum_i^{-1} \mu_j - 2\mu_i^T \textstyle\sum_i^{-1} \mu_j], \tag{10}$$

where μ is the centroid of a cluster, Σ_f^{-1} is the inverse of the covariance of each cluster, and *T* is the transpose. Step (3) Find two clusters that have the shortest distance. And then, step (4) update the new cluster by calculating a member N_{new}, centroid μ_{new} and $\sum_{new}$ New covariance using equation (12-14).

$$N_{new} = N_i + N_j\,, \tag{11}$$

$$\mu_{new} = \frac{N_i}{N_{new}}\mu_i + \frac{N_j}{N_{new}}\mu_j\,, \tag{12}$$

$$\sum new = \frac{N_i - 1}{N_{new} - 1}\sum i + \frac{N_j - 1}{N_{new} - 1}\sum j + \frac{N_i N_j}{N_{new}\,(N_{new} - 1)}\left[\left(\mu_i - \mu_j\right)\left(\mu_i - \mu_j\right)^T\right], \tag{13}$$

Where i and j are two clusters measured distance. Step (5) to reduce the number of centroids, repeat steps 2 through 5 until the maximum number of clusters c has been achieved. Use equation (14) to calculate the value of α, where the distance d_{ij} Is the distance between pixels x_i to the centroid μ_j and Σ_j^{-1} is the inverse of the covariance matrix of cluster *j* calculated using equation (15).

$$a_j = \frac{\sum_{i=1}^{N} d_{ij}}{N}, \quad (14)$$
$$d_{ij} = (x_i - \mu_j)^T \textstyle\sum_j^{-1} (x_i - \mu_j) \quad (15)$$

Based on the above steps, the result of the cluster is achieved as the covariance matrix of each cluster and alpha values.

2.2 Clustering Pixels with Mahalanobis Fuzzy C-Means Spatial Information

In color images, neighboring pixels are usually interdependent and correlate strongly. But, MFCM [12] only considers the color space situation, not the neighborhood system of the image. It was the effect of edge detection processes that performed unwell and generated less accuracy in segmentation results. Therefore, we included the local spatial interaction between adjacent pixels in the fuzzy membership function. If the adjacent pixels have similar properties, the central pixel will likely be grouped into the same cluster as the adjacent pixels.

Let$X = \{x_i : i = 1, 2, \ldots , N\}$ be the observation image, where i is in response to the pixel index, $x_i = \{x_{i1}, x_{i2}, \ldots, x_{id}\}^T$ It is the pixel element vector, d is the pixel size, and N is the numeral of pixels in the picture X. The factual procedure of the FCM algorithm can be written in the following formula (16)

$$J_{fcm} = \sum_C^N \sum_{j=1}^{c} u_{ij}^m d_{ij} \quad (16)$$

Where c is the number of clusters, j is the clustered index, and $\boldsymbol{U}$ = $[\mu_{ij}]$ $N \times c$ is the membership matrix representing the fuzzy segment, u_{ij} Is the degree of association where xi belongs to the jth cluster and satisfies the represents c_j=1 u_{ij} = 1. The fuzzy coefficient m is the weighted index of u_{ij} and represents the degree of ambiguity in the algorithm, $d_{ij} = ||x_i - u_j||_2$ represents the Euclidean distance representing the difference between the pixel vector x_i and the mean The vector of the jth cluster measuring $\boldsymbol{\mu_j = (\mu_{j1}, \mu_{j12}, \ldots\ldots \mu_{jd})^T}$.

Zhao [12] proposed to use Mahalanobis distance for measuring the dissimilarity between the pixel vector x_i and the mean vector of the jth cluster because the Euclidean distance as a measure of dissimilarity indicates that it's susceptible to noise and cluster differences. Therefore, the Mahalanobis space represents the dissimilarity of clusters that differ more accurately than the Euclidean distance. The membership functions and the cluster center are written in formulas 17 and 18:

$$u_{ij} = \frac{\alpha j \, exp(-\frac{d_{ij} + \lambda log \log |\sum j|}{\lambda})}{\sum_{j'=1}^{c} \alpha j' exp(-\frac{d_{ij} + \lambda log \log |\sum j|}{\lambda})} \quad (17)$$

Where u_{ij} represents the membership of pixel x_j in the ith cluster, α_j is a variable to control cluster size, Σ_j It is the covariance matrix of the jth cluster, and λ is the degree of the fuzziness of the algorithm.

find values μ_j centroid using equation

$$\mu_j = \frac{\sum_{i=1}^{N} u_{ij} x_i}{\sum_1^N u_{ij}} \quad (18)$$

Where N is the number of data, u_{ij} Is the degree of membership of pixels ith to cluster j and x_i Is the feature vector data ith. Looking Σj covariance matrix for each cluster j using the equation:

$$\sum j = \frac{\sum_{i=1}^{N} u_{ij}(x_i - \mu_j)(x_i - \mu_j)^T}{\lambda \sum_{i=1}^{N} u_{ij}} \qquad (19)$$

The centroid value and covariance matrix for each cluster will be used as a reference to determine the distance of data to the centroid. Measuring dissimilarities of each pixel towards each centroid to determine how close the pixel data x_i against cluster centroid μ_j using Mahalanobis distance as shown in the equation

$$d_{ij} = (x_i - \mu j) \sum_j^{-1} (x_i - \mu j) \qquad (20)$$

where d_{ij} Is the distance between the centroid of pixels x_i, μ_j and $\sum_j^{-1}$ is the inverse of the covariance matrix cluster *j/* Calculate the value of alpha α_j Where α_j is a variable that controls the size of the cluster, α_j Calculated using equation (22).

$$\alpha_j = \frac{\sum_{i=1}^{N} uij}{N} \qquad (21)$$

One of the crucial characteristics of images is that they have a high correlation between adjacent pixels. In other words, the higher the correlation the neighboring pixels perform, the greater the image clusters. These spatial relationships are essential in clustering but are not implemented in the standard FCM algorithm. We make use of spatial information, and the computation of spatial objects is described as follows:

$$h_{ij} = \sum_{k \in NB(x_j)} u_{ik} \qquad (22)$$

A 3x3 window was best implemented by the $NB(x_j)$ whose center was on a pixel $x_j$in the spatial domain, Like the membership function, it was determined in the spatial function h_{ij} that the pixel x_j Probability was on ith cluster. For a cluster, the spatial function of a pixel is considered significant when most of its surroundings fit within the same cluster. The membership function below is an amalgamation of spatial functions.

$$u'_{ij} = \frac{u_{ij}^p h_{ij}^q}{\sum_{k=1}^{c} u_{kj}^p h_{kj}^q} \qquad (23)$$

the relative importance of both functions, h_{ij} and u'_{ij}, is controlled by the parameters of p and q. In a homogenous region, the clustering result does not generate any transformation, and the spatial function is a fortress of the original property.

The proposed algorithm process can be summarized as follows:

Algorithm MFCMS

Input: level of obscurity m = 2, p = 2, q = 1, λ=1.5, t=0, T = 100 and the error ε

1. Initialize the center μ_j^0 , covariance $\sum j$, alpha αj cluster from phase 1(HTM)
2. Repeat
3. **if t =0**
   a. Calculate the membership value

   $$u_{ij} = \frac{\alpha j \, exp(-\frac{d_{ij} + \lambda log \log|\sum j|}{\lambda})}{\sum_{j'=1}^{c} \alpha j' exp(-\frac{d_{ij} + \lambda log \log|\sum j|}{\lambda})}$$

   end

   else

   a. Update the new centroid of the clusters $\mu^{(t)} = \{\mu_1^{(t)}, \mu_2^{(t)}, .., \mu_c^{(t)}\}$ as follows:

   $$\mu_j = \frac{\sum_{i=1}^{N} u_{ij} x_i}{\sum_1^N u_{ij}}$$

b. Update covariance matrices of the clusters $\sum^{(t)} = \{\sum_{(1)}^{(t)}, \sum_{(2)}^{(t)}, \ldots, \sum_{(c)}^{(t)}\}$ as follows:

$$\sum j = \frac{\sum_{i=1}^{N} u_{ij}(x_i - \mu_j)(x_i - \mu_j)^T}{\lambda \sum_{i=1}^{N} u_{ij}}$$

c. Calculate Mahalanobis distance pixel of cluster d_{ij} As follows:

$$d_{ij} = (x_i - \mu j) \sum_j^{-1} (x_i - \mu j)$$

d. Calculate the variable $\alpha^t = \{\alpha_1^{(t)}, \alpha_2^{(t)}, \ldots, \alpha_c^{(t)}\}$

$$\alpha_j = \frac{\sum_{i=1}^{N} uij}{N}$$

e. Calculate the membership matrix $U^{(t+1)} = [u_{ij}^{t+1}]Nxc$ as follows:

$$u_{ij} = \frac{\alpha j\, exp(-\frac{d_{ij} + \lambda log \log|\sum j|}{\lambda})}{\sum_{j'=1}^{c} \alpha j' exp(-\frac{d_{ij} + \lambda log \log|\sum j|}{\lambda})}$$

f. Calculate the Spatial Membership value $U'^{(t+1)} = [u'^{t+1}_{ij}]Nxc$ as follows:

$$u'_{ij} = \frac{u_{ij}^p h_{ij}^q}{\sum_{k=1}^{c} u_{kj}^p h_{kj}^q}$$

Where $h_{ij} = \sum_{k \in NB(x_j)} u_{ik}$

end

4. Until $\| u_{ij}^t - u_{ij}^{t-1} \| < \varepsilon$ or iteration t = T, then stop
Otherwise, t= t+1 and go to step 2.

Determine the membership of each cluster, then specify the cluster that represents the object or background. If the number of clusters $c > 2$, join the cluster background to obtain two clusters, each representing the object area and the background using equation (7).

3. RESULTS

We implemented our experiments based on the Weizmann Segmentation Dataset to investigate the proposed method. Ten images are applied with a size of 217 x 387 pixels in this implementation. Those pictures are chosen to display the ability of the proposed MFCMS strategy. Our proposed method uses MATLAB 2015b, which runs on a computer with a Pentium i3 2.30 Hz CPU and 4.00 GB of RAM. The experiment performance result of the proposed method is visually shown in Figure 2. We use Misclassification Error (ME) to calculate the percentage of pixel errors in the image, which we compare with the ground truth experts have made [13]. ME can be easily expressed as:

$$ME = 1 - \frac{|B_0 \cap B_t| + |F_0 - F_T|}{|B_t| + |F_0|} \quad (24)$$

Where background and foreground are indicated by B_0 and F_0 for the actual picture and by B_T and F_t For the test picture. Foreground area relative error (RAE) [14], [13], [15], The reference picture is compared to the segmentation result to the range of mismatches withinside the segmented picture is measured. It is described as the following formula.

$$RAE = \frac{D_0 - D_r}{D_0} \; if \; D_r < D_0$$

$$\frac{D_r - D_0}{D_r} \quad if\ D_0 < D_r \qquad (25)$$

Where D_r is the reference image area, and D_0 is the area of the threshold image.

ME and RAE values of segmented images compared to actual images of the proposed method. The Smaller ME and RAE values result in better performance (see Table 1) for the proposed method.

Image 1

Image 2

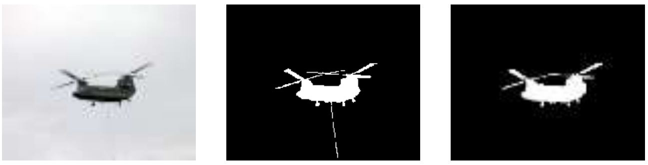

Image 3

Image 4

Image 5

Image 6

Image 7

Image 8

Image 9

Image 10

Figure 2 Examples of image segmentation results. First column: original images. Second column: ground truth. Third column: MFCMS Segmentation Result

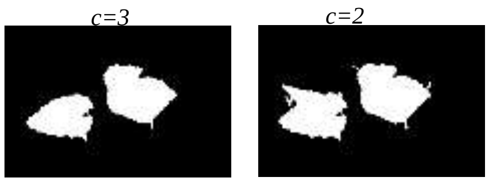

Figure 3. Determination of the number of clusters that affect segmentation results

4. DISCUSSION

4.1 Segmentation Result

The proposed method of incorporating spatial information and space color features on MFCMS produces good accuracy. Based on Table 1, image 2 performs the best segmentation result by the value of ME and RAE are equal to 0.61% and 0.67%. On the contrary, Image 4 displays poor accuracy results because the impact of color intensity and variance is very influential on the proposed method. The higher difference in both color intensity and variance of the object and the background color provides higher accuracy of segmentation results.

Table 1. The result of the segmented image

Images	ME (%)	RAE(%)
Image1	1.07	**0.44**
Image2	**0.61**	0.67
Image3	2.1	9.93
Image4	2.17	10.2
Image5	1.21	3.96
Image6	1.14	1.19
Image7	3.64	1.41
Image8	0.72	0.74
Image9	1.66	3.26
Image10	1.8	3

4.2 Parameter Setting

The weighted membership function of the parameters p and q has a significant impact on the value of the last section u_{ij} and cluster center μ_j . Hence, the value of the last section u_{ij} and cluster center μ_j Also, give an impact on the final result of segmentation. Table 2 describes the average performance of the MFCMS algorithm for all images in ME and RAE by varying values of p and q using a fixed neighborhood $NB(x_j)$ of size 3 × 3. The MFCMS algorithm uses p = 2 and q = 1 to provide excellent results. The results show that neighborhood and worldwide club values need to focus on the convolution technique while producing the final club value and cluster center because it makes the membership function equally important.

The result of image segmentation has a strong correlation between the neighboring pixels. Table 3 describes the performance of the csFCM algorithm when scaling neighboring $NB(x_j)$ using p = 2 and q = 1. The algorithm gives good results for $NB(x_j)$ = 3 × 3. Subsequent experiments are performed using p = 2, q = 1 and $NB(x_j)$ = 3 × 3 because the results help the previous claim about the correlation between adjacent pixels.

Table 2. the results of experimental comparisons on parameters *p* and *q*

Parameter		Evaluation Method	
P	Q	ME	RAE
1	2	2.11	2.17
2	1	**1.61**	**3.48**
2	2	2.11	2.12

Table 3. comparison result experiment on the neighborhood size parameter

Parameter	Evaluation Method

Neighborhood	ME	RAE
3x3	**1.61**	**3.48**
5x5	1.79	4.00
7x7	1.90	4.11

4.3 Merging Cluster

Determining the number of clusters c on the pixel clustering (MFCMS) step also dramatically influences the segmentation process. Determining the amount of the initial cluster before the pixel clustering stage, we should know the proper number of dominant colors in the image, as illustrated in image 10 of Figure-3. It can be seen that the segmentation results with cluster = 3 can be appropriately segmented, but cluster = 2 instead occurs over-segmentation.

Our proposed method depends on the color variant in the image. Both different and similar variant colors between the object and the background are delicate for one object image. However, for the image of two objects or more, the variant colors of the objects have to be likely similar to each other and contrast with the background. If the object's color is prone to be dissimilar to other objects, there will be an over-segmentation of the result. So, the methods can only segment likely similar variant colors of two or more objects.
Determination of the number of clusters c is also very influential on the method, so a method is needed to determine the optimal amount of c.

5. CONCLUSIONS

In this paper, we proposed a new method Mahalanobis Fuzzy C-mean with Spatial Information (MFCMS). The critical factor of this method is to define additional spatial space in MFCM using spatial facts through the last club feature, which can increase the accuracy of segmentation results considering color space and spatial space. Based on the evaluation of the images obtained, MFCMS receives higher segmentation overall performance with misclassification error (ME) and relative foreground area error (RAE) values of 1.61 and 3.48.

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