Steady State and Dynamic Interaction Analysis in Multivarible Control System

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Relative gain array (RGA) and average dynamic gain array (ADGA) have been investigated as a measure for interaction of multi-input multi-output (MIMO) 2x2 system. Several examples have been chosen to represent MIMO with various time constants and dead time of first order plus dead time. The ratio of off-diagonal and on-diagonal element time constants of the transfer function processes ($\hat{o}_{ij}$) has shown more dominance to interaction than ratio of off-diagonal and on-diagonal element dead times of the transfer function processes ($\ddot{a}_{ij}$) . RGA, INA, and ADGA give the same result for t_{ij} ³ 1 and INA and ADGA should be used for $t_{ij} < 1$ regardless the value of d_{ij}.

Keywords: Steady state gain, dynamic interaction analysis and measurement, control pairing, relative gain array (RGA), inverse Nyquist array (INA), average dynamic gain array (ADGA), *and* multi-input multi-output (MIMO) process.

INTRODUCTION

Relative gain array or *RGA* (Bristol 1966) is a relatively easy method of interaction measurement. Calculated simply by using steady state process gain, RGA can give information on the selection of control pairing. Unfortunately, RGA often gives wrong indication when a system has a significant interaction (Jensen et al. 1986), especially in a situation where dynamic characteristic is an important factor in the control configuration.

Therefore, other methods have been developed for interaction measurement (Rosenbrock 1969, Tung et al. 1977, Witcher et al. 1977) where the dynamic effect on the process was taken into account. These methods have successfully been applied by several workers (Gagnepain et al. 1982, Gede et al. 1997).

In this paper, RGA will be studied to find out how far it can be used as a measure of the significance of dynamic interaction in multivariable processes. The results of using methods of dynamic interaction measurement, such as inverse nyquist array or INA (Rosenbrock 1969) and average dynamic gain array or ADGA (Gagnepain et al. 1982), will be compared with those of RGA.

REVIEW OF SOME INTERACTION MEASUREMENTS

Steady state interaction measurement: The Relative Gain Array (RGA)

RGA, a matrix of numbers, λ_{ij}, is defined as the ratio of the steady state gain between *i*-th controlled variable and *j*-th manipulated variable with all other manipulated variables kept constant, relative to the steady state gain between the same two variables with all the other controlled variables kept constant.

$$\lambda_{ij} = \text{open loop / closed loop gain} \quad (1)$$

The open loop gain between y_i and u_j is K_{ij}, which is the element of the process gain matrix *K*. The closed loop gain matrix is $1/\tilde{K}_{ij}$, where $\tilde{K}_{ij}$ is the element of the transpose of K^{-1} matrix. Hence, the relative gain (l_{ij}) can be expressed as:

$$\lambda_{ij} = K_{ij}\tilde{K}_{ij} \quad (2)$$

Dynamic interaction measurement

- ***Inverse Nyquist Array (INA)***

  This INA method gives information on system stability in addition to indicating process interaction by looking at the radii of the Gershgorin circles.

  Consider the equation

$$\boldsymbol{Q}(i\omega) = \boldsymbol{G}(i\omega)\,\boldsymbol{G}_c(i\omega) \quad (3)$$

  Where $\boldsymbol{G}(iw)$ and $\boldsymbol{G}_c(iw)$ are the process transfer function and controller matrix, respectively, and $\boldsymbol{Q}(iw)$ is the product of these two matrices. Let $Q(iw)$ and q_{ij} denote the inverse matrix of $\boldsymbol{Q}(iw)$ and the element of inverse matrix of $\boldsymbol{Q}(iw)$. The radii of Gershgorin circles are defined as the absolute value of the off-diagonal element in this inverse matrix, $\hat{Q}(iw)$, calculated at a certain frequency. If the elements of off-diagonal are zero, then radii of Gershgorin circles equal zero. This means no interaction is present in the system. The larger the radii, the larger the interaction in the system. Therefore, the choice of open loop pairing is one that gives the smallest radii of the Gershgorin circles.

- ***Average Dynamic Gain Array (ADGA)***

  Average Dynamic Gain Array is a matrix of numbers, μ_{ij}, and is an interaction measurement developed by introducing frequency response to the open loop transfer function (Gagnepain et. al 1982) from the concept of Relative Dynamic Array (Witcher et. al. 1977) based on open loop step response.

  Let the process model for open loop system be given by

$$\boldsymbol{y}(s) = \boldsymbol{G}(s)\,\boldsymbol{u}(s) \quad (4)$$

where each transfer function is expressed as first order plus dead time (FOPDT).

$$G(s) = \frac{K_r e^{-ds}}{Ts+1}$$

Initially, the process was on the steady state condition ($\boldsymbol{u} = \boldsymbol{y} = 0$) and that a unit step change in u_j occurs at $t = 0$. During dead time interval $[0, d_{ij}]$, y_i is not affected by u_j. For time interval $[d_{ij}, \theta]$, average dynamic gain, D_{ij} between y_i and u_j can be calculated as

$$D_{ij} = (\text{average change in } y_i)/(\text{change in } u_j) \quad (5)$$

$$D_{ij} = \frac{1}{\theta - \theta_1}\int_{\theta_1}^{\theta} y_i(t)\,dt \quad (6)$$

The element of ADGA is defined as:

$$\mu_{ij} = D_{ij}(t)D_{ij}(t) \quad (7)$$

where:

$$\theta_1 = \min\{\max(d_{11}, d_{22}), \max(d_{12}, d_{21})\} \quad (8)$$

$$\theta = \theta_1 + T_M \quad (9)$$

T_M = largest time constant in the process transfer function matrix

The control pairing recommended by average dynamic gain array is the same as recommended by relative gain array, that is for μ_{11}<0.5, than u_1 is paired with y_2 while for μ_{11}>0.5 than u_1 is paired with y-$_1$.

CASE STUDIES

Proportional integral controller was used for controlling all the processes in this paper. The controller parameters were found by Biggest Log-Modulus Tuning method (Luyben 1986). A criterion of preferred control pairing is the IAE.

Nine examples (see Table 1) of 2x2 transfer function matrices whose first order plus dead time models as their elements were studied in this paper, where all the matrices had identical steady-state process gain, that is:

$$K_p = \begin{bmatrix} -2 & 1.5 \\ 1.5 & 2 \end{bmatrix}; \lambda = 0.64$$

However, time constant and dead times varied. These transfer function models were chosen for describing the effect of process dynamic control pairing. For the control parameters to be tuned easier, the models were given symmetrically, G_{11} = -G_{22} and G_{12} = G_{21}. These models can be classified as

τ < 1	δ < 1	Example 8
	δ = 1	Example 7
	δ > 1	Example 9
τ = 1	δ < 1	Example 5
	δ = 1	Example 4
	δ > 1	Example 6
τ > 1	δ < 1	Example 2
	δ = 1	Example 1
	δ > 1	Example 3

Table 1. The Transfer Function Models Studied

Example 1 $\mu_{11} = 0.81$

$$G(s) = \begin{bmatrix} \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{20s+1} \\ \frac{1.5s^{-s}}{20s-1} & \frac{2e^{-s}}{10s+1} \end{bmatrix}$$

Example 2 $\mu_{11} = 0.79$

$$G(s) = \begin{bmatrix} \frac{-2e^{-3s}}{10s+1} & \frac{1.5e^{-s}}{20s+1} \\ \frac{1.5s^{-s}}{20s+1} & \frac{2e^{-3s}}{10s+i} \end{bmatrix}$$

Example 3 $\mu_{11} = 0.83$

$$G(s) = \begin{bmatrix} \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-3s}}{20s+1} \\ \frac{1.5s^{-3s}}{20s+1} & \frac{2e^{-s}}{10s+1} \end{bmatrix}$$

Example 4 $\mu_{11} = 0.64$

$$G(s) = \begin{bmatrix} \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-s}}{10s+1} \\ \frac{1.5s^{-s}}{10s+1} & \frac{2e^{-s}}{10s+1} \end{bmatrix}$$

Example 5 $\mu_{11} = 0.51$

$$G(s) = \begin{bmatrix} \frac{-2e^{-3s}}{10s+1} & \frac{1.5e^{-s}}{10s+1} \\ \frac{1.5s^{-s}}{10s+1} & \frac{2e^{-3s}}{10s+1} \end{bmatrix}$$

Example 6 $\mu_{11} = 0.71$

$$G(s) = \begin{bmatrix} \frac{-2e^{-s}}{10s+1} & \frac{1.5e^{-3s}}{10s+1} \\ \frac{1.5s^{-3s}}{10s+1} & \frac{2e^{-s}}{10s+1} \end{bmatrix}$$

Example 7 $\mu_{11} = 0.43$

$$G(s) = \begin{bmatrix} \frac{-2e^{-s}}{20s+1} & \frac{1.5e^{-s}}{10s+1} \\ \frac{1.5s^{-s}}{10s+1} & \frac{2e^{-s}}{20s+1} \end{bmatrix}$$

Example 8 $\mu_{11} = 0.39$

$$G(s) = \begin{bmatrix} \frac{-2e^{-3s}}{20s+1} & \frac{1.5e^{-s}}{10s+1} \\ \frac{1.5s^{-s}}{10s+1} & \frac{2e^{-3s}}{20s+1} \end{bmatrix}$$

Example 9 $\mu_{11} = 0.45$

$$G(s) = \begin{bmatrix} \frac{-2e^{-s}}{20s+1} & \frac{1.5e^{-3s}}{10s+1} \\ \frac{1.5s^{-3s}}{10s+1} & \frac{2e^{-s}}{20s+1} \end{bmatrix}$$

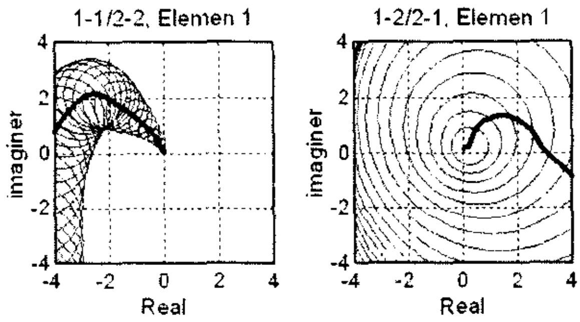
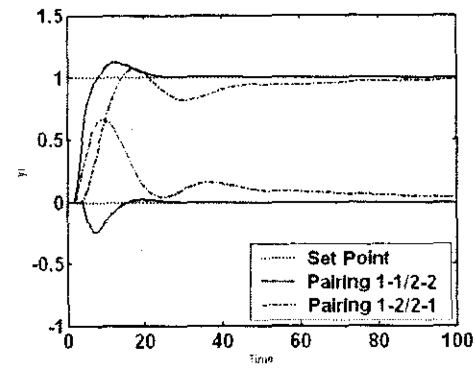


Figure 3. (a) The INA Plot for Example 3



(b) Closed Loop Response for Example 3

where τ and δ are defined by

$$\tau = \frac{T_{12}T_{21}}{T_{11}T_{22}} \tag{10}$$

$$\delta = \frac{d_{12}d_{21}}{d_{11}d_{22}} \tag{11}$$

Each of the nine examples has a value for the relative gain of 0.64; therefore, according to RGA, one has to use the pairing controller 1-1/2-2. Based on the ADGA and INA, even with a relative gain of 0.64, there are several examples (Examples 7, 8, and 9) that one has to make use of the pairing controller 1-2/2-1, instead of using pairing controller 1-1/2-2, due to the value of μ_{11} is less than 0.5 (less interaction if the control pairing is 1-2/2-1) and due to the smaller radii Gershgorin circles. Therefore, RGA should be taken cautiously before determining the method for control pairing.

Examples 1–3 show that for $\tau > 1$, both steady state and dynamic interaction measurements (RGA and ADGA) suggest 1-1/2-2 pairing as λ_{11} and μ_{11} are larger than 0.5. Figures 1a, 2a, and 3a indicate that less interaction occurs for control pairing 1-1/2-2 due to the smaller radii of Gershgorin circles. Therefore, from the INA plot, it can be concluded that less interaction occurs when one uses 1-1/2-2 pairing than 1-2/2-1 pairing. The closed loop responses for these examples are shown on figures 1b, 2b, and 3b, respectively, where the 1-1/2-2 pairing gives better responses than the 1-2/2-1 pairing, indicated by their faster rise time and settling time, and less oscillating. The IAE values for each possible pairing for each models are listed in Table 2. They also justify that 1-1/2-2 pairing gives smaller IAE value than 1-2/2-1 pairing. Therefore, for $\tau > 1$ the interaction analysis for these models, the steady state interaction measurement is deemed sufficient.

Table 2. Simulation Results for $\tau > 1$

System	Pairing	IAE Value		Loop 1		Loop 2	
		Loop 1	Loop 2	*Kc*	τ_I	*Kc*	τ_I
Example 1	1-1/2-2	4.24	1.52	-1.44	8.22	1.44	8.22
	1-2/2-1	9.22	9.54	1.01	31.03	1.01	31.03
Example 2	1-1/2-2	8.01	5.27	-0.74	16.14	0.74	16.14
	1-2/2-1	14.65	14.84	0.77	40.65	0.77	40.65
Example 3	1-1/2-2	4.92	1.67	-1.56	7.56	1.56	7.56
	1-2/2-1	12.75	14.36	0.78	40.42	0.78	40.42

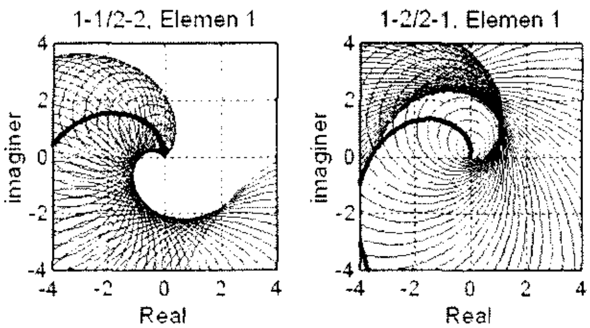
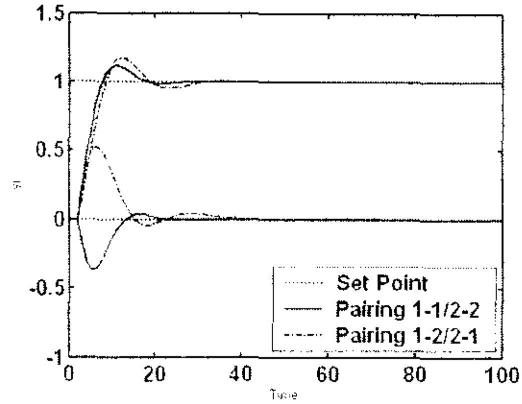


Figure 4. (a) The INA Plot for Example 4



(b) Closed Loop Response for Example 4

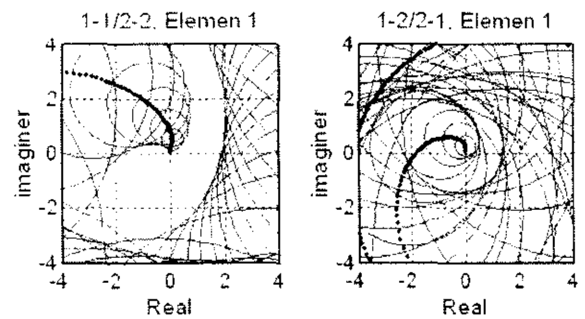
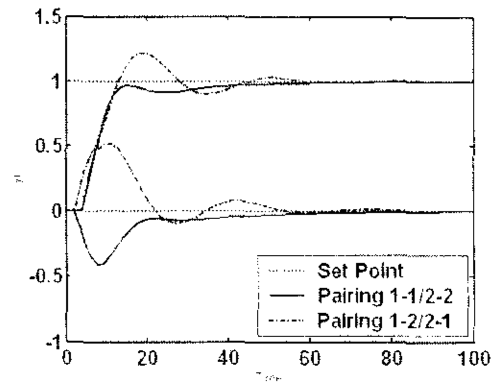


Figure 5. (a) The INA Plot for Example 5



(b) Closed Loop Response for Example 5

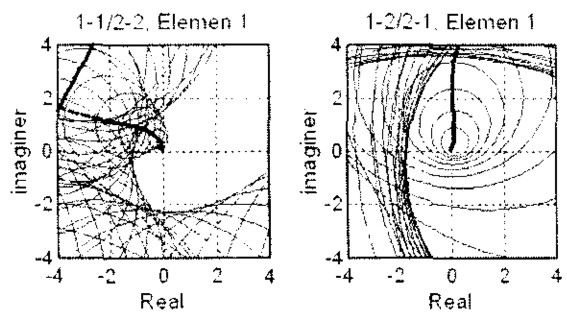
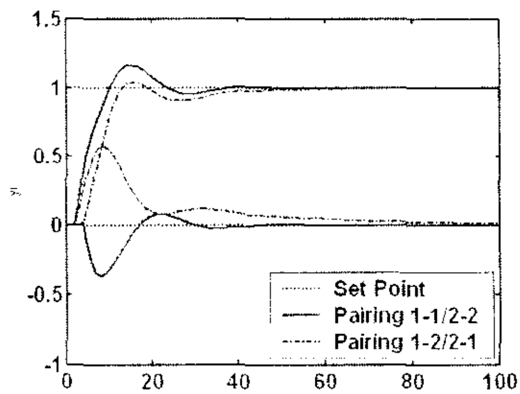


Figure 6. (a) The INA Plot for Example 6



(b) Closed Loop Response for Example 6

For examples 4 and 6, both steady state and dynamic interaction measurements strongly suggest 1-1/2-2 pairing, whereas for example 5, ADGA only "slightly" recommends 1-1/2-2 pairing, since $\mu_{11} = 0.51$, which is close to 0.5. Figures 4a, 5a, and 6a show that control pairing should be 1-1/2-2 as given by the INA plot, since it gives less interaction. However, inspecting the transient responses of these models in figures 4b, 5b, and 6b, one concludes that all 1-1/2-2 pairing suggested by RGA has faster rise time and settling time, smaller overshoot, less oscillation, and gives smaller IAE values than 1-2/2-1 pairing as can be seen in Table 3. Therefore, for $\tau = 1$, the interaction analysis for these models are again sufficient to merit the use of the steady state interaction measurement only.

Table 3. Simulation Results for $\tau = 1$

System	Pairing	IAE Value		Loop 1		Loop 2	
		Loop 1	Loop 2	*Kc*	τ_I	*Kc*	τ_I
Example 4	1-1/2-2	4.17	2.50	-1.17	10.07	1.17	10.07
	1-2/2-1	5.58	4.76	1.07	14.72	1.07	14.72
Example 5	1-1/2-2	8.83	6.63	-0.66	18.16	0.66	18.16
	1-2/2-1	9.63	7.74	0.91	17.38	0.91	17.38
Example 6	1-1/2-2	5.92	3.66	-1.12	10.54	1.12	10.54
	1-2/2-1	8.53	10.35	0.69	23.02	0.69	23.02

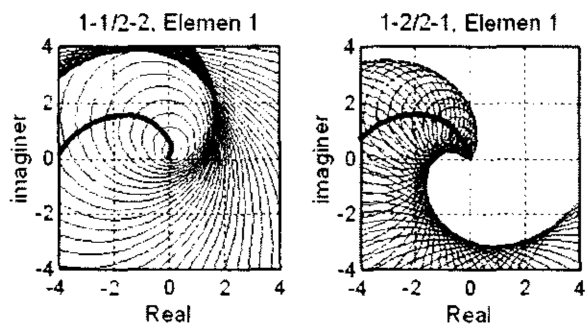
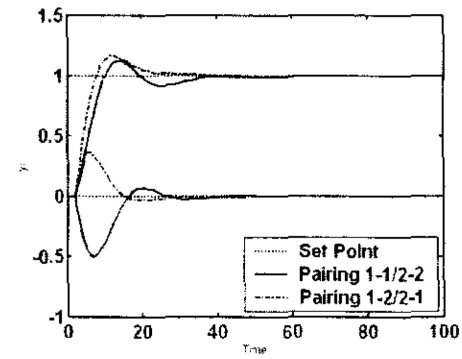


Figure 7. (a) The INA Plot for Example 7



(b) Closed Loop Response for Example 7

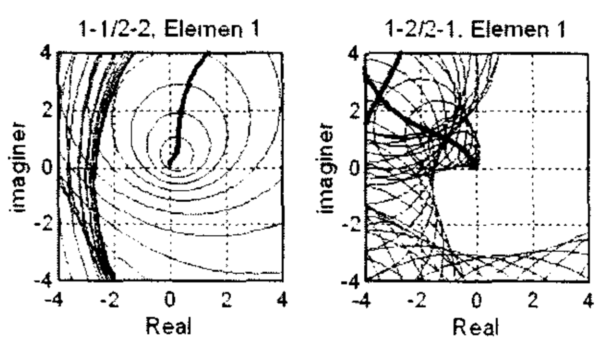
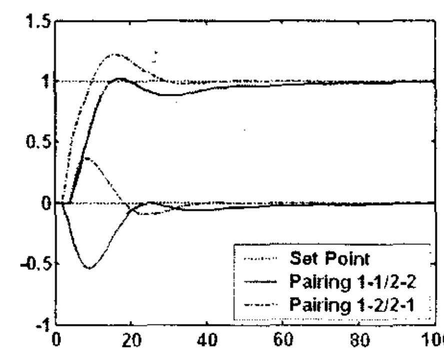


Figure 8. (a) The INA Plot for Example 8



(b) Closed Loop Response for Example 8

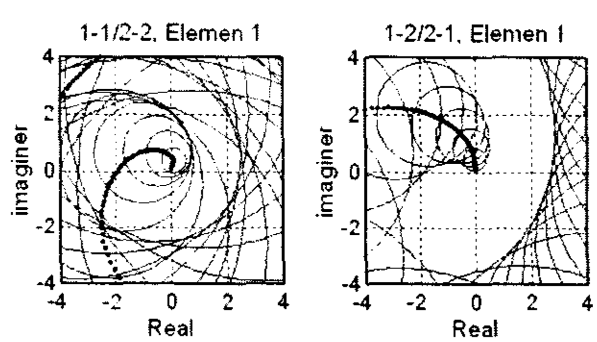
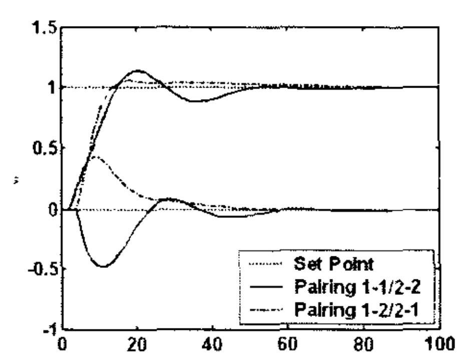


Figure 9. (a) The INA Plot for Example 9



(b) Closed Loop Response for Example 9

Table 4. Simulation Results for $\tau = 1$

System	Pairing	IAE Value		Loop 1		Loop 2	
		Loop 1	Loop 2	*Kc*	τ_I	*Kc*	τ_I
Example 7	1-1/2-2	6.65	4.85	-1.22	19.33	1.22	19.33
	1-2/2-1	5.30	3.19	1.57	10.02	1.57	10.02
Example 8	1-1/2-2	10.83	7.88	-0.83	28.38	0.83	28.38
	1-2/2-1	6.67	4.33	1.57	10.04	1.57	10.04
Example 9	1-1/2-2	10.19	7.37	-0.98	24.07	0.98	24.07
	1-2/2-1	8.64	6.92	0.88	18.09	0.88	18.09

For examples 7, 8, and 9, all of the control pairings suggested by the RGA, that is the 1-1/2-2 pairing, are strongly different from that suggested by ADGA and INA, where both INA and ADGA suggest the 1-2/2-1 pairing. Figures 7a, 8a, and 9a show that the Gershgorin circles have radii smaller for 1-2/2-1 pairing than for 1-1/2-2 pairing. Figure 7b shows that both 1-1/2-2 and 1-2/2-1 pairings, for example 7, give the responses that are somewhat similar. However, the response of 1-2/2-1 pairing is a bit faster and has smaller overshoot compared to the response of 1-1/2-2 pairing. Even though the response of 1-2/2-1 pairing for example 8, as shown on Figure 8b, has higher overshoot for upper loop than that of 1-1/2-2 pairing for the same loop, but on the contrary, its lower loop has smaller overshoot. In general, 1-2/2-1 pairing has faster response compared to 1-1/2-2 pairing. From Figure 9b, it is clear that 1-2/2-1 pairing gives better response than 1-1/2-2 pairing. The IAE values for each response for examples 7, 8, and 9 are shown in Table 4, which verify that 1-2/2-1 pairing is better than 1-1/2-2 pairing. Hence, RGA fails to recommend the proper pairing for the last-three cases. Therefore, for model that has $\tau < 1$, one has to use dynamic interaction measurements, such as ADGA and INA, to characterize the system interaction accurately, since the steady state RGA may give misleading recommendations on controller pairing.

CONCLUSIONS

From this work, it can be concluded that especially for multivariable processes, interaction measurements resulting from the RGA method provide satisfactory control pairing only for certain cases. In these studies it provides good control pairing just for the system that has $\tau > 1$ and $\tau = 1$. For the system that has $\tau < 1$, the dynamic behavior tends to have a larger process interaction and, therefore, one has to use dynamic interaction measurements, such as ADGA or INA, as shown in this study.

Further studies on the effect of process gain on the transfer function models are being carried out including the extention of the 2x2 system to to the 3x3 system.

ACKNOWLEDGMENT

The authors wish to thank The National Research Council of Indonesia (Dewan Riset Nasional) for the financial support given to the project through the RUT V Grant No. 226/SP/RUT/BPPT/IV97 of 17 April 1997.

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