Comparison of Steady State and Dynamic Interaction Measurements in Multiloop Control Systems

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The applicability of the steady-state Relative Gain Array (RGA) to measure dynamic process interactions in a multiloop control system was investigated. Several transfer function matrices were chosen, and the gains, time constants, and dead times of their elements were varied to represent the systems with dominant dynamic interactions. It was shown that the steady-state RGA method predicted the controller pairing accurately if the pairing elements recommended by RGA had the bigger gains and the same or smaller time constants compared to other elements in the corresponding rows. When these conditions were not met, the RGA would give a wrong result, and dynamic interaction measurements, such as the Average Dynamic Gain Array (ADGA) and the Inverse Nyquist Array (INA), should be used instead to determine the best controller pairing in a multiloop control system.

Keywords: Control pairing, dynamic process interaction, multiloop control systems, Relative Gain Array (RGA), *and* steady state.

INTRODUCTION

Multivariable control occurs in nearly all industrial processes because flow rate, inventory, temperature, and product quality need to be controlled simultaneously. The multiloop approach was the first approach used for multivariable control in the process industries. However, it is still widely used due to its success through the decades.

There are several advantages offered by the multiloop strategy. Firstly, it uses simple algorithms that can be implemented easily with inexpensive analog computing equipment. Secondly, it has a simple control structure that can be understood easily by the plant operator. Thirdly, the standard control designs based on the multiloop strategy have been widely developed for common unit operations. Therefore, multiloop designs will continue to be used extensively although not exclusively. (Marlin 2000)

The design and operation of multivariable processes are typically more complex and more difficult than single-input single-output (SISO) processes because of the interactions between input and output variables. Hence, the key design decision is to determine the proper pairing of controlled variables with manipulated variables. If incorrect pairing is used, the designed control system may perform very poorly or even become inoperable. (Shinskey 1996) Therefore, methods to measure process interactions have been developed.

Undoubtedly, the most widely used approach for measuring process interactions is the Relative Gain Array (RGA) proposed by Bristol (1966). This approach is the easiest way to characterize interactions because it requires only steady-state process information. Hence, it is also the approach usually discussed in textbooks (Ray 1994, Shinskey 1996, Marlin 2000). RGA, however, often fails to indicate when a system has significant interaction problems and may give misleading recommendations on controller pairing. It has been suspected that these limitations are due to the absence of process dynamics in RGA. Consequently, other measures for process interactions have been proposed which include both static and dynamic process information. Three of these methods are the Inverse Nyquist Array or INA (Rosenbrock 1969), the Relative Dynamic Array or RDA (Witcher and McAvoy 1977), and the Average Dynamic Gain Array or ADGA (Gagnepain and Seborg 1982).

Handogo et al. (2004) studied the use of these three methods for a 2x2 system having first order plus dead time (FOPDT) processes. Unfortunately, the study was limited to a constant ratio of steady-state gains of less than 1.

Considering RGA's simplicity, wider range of utilizations, and limitations, the present research investigated the extent to which the method can be used to (a) measure process interactions and (b) determine the correct controller pairing in multiloop control systems.

The investigations started with 2x2 systems and the results were generalized for NxN systems. The steady state gains, time constants, and dead times of the elements of the transfer function matrix (TFM) were varied; hence, controller pairings were predicted using the RGA, ADGA, and INA methods. The transient responses of the control configurations suggested by each method were then compared through closed loop simulations using MATLAB™ and SIMULINK™.

REVIEW OF SOME INTERACTION MEASUREMENTS

Relative Gain Array (RGA)

To review the RGA method, consider the open loop multivariable process shown in Figure 1. The relative gain between the *ith* controlled variable C_i and the *jth* manipulated variable M_j denoted by λ_{ij} is defined as:

$$\lambda_{ij} = \frac{\left[\left(\Delta C_i / \Delta M_j \right)_{\Delta M_k = 0} ; k \neq j\right]}{\left[\left(\Delta C_i / \Delta M_j \right)_{\Delta C_k = 0} ; k \neq i\right]} \qquad (1)$$

Bristol (1966) had shown that to evaluate the RGA $\underline{\underline{\lambda}}$, which is the matrix with the elements of λ_{ij}, only the open loop gain would be necessary.

$$\underline{\underline{\lambda}} = \underline{\underline{G(0)}} \, \mathrm{O} \left[\underline{\underline{G(0)}}^{-1} \right]^{T} = \underline{\underline{K}} \, \mathrm{O} \left[\underline{\underline{K}}^{-1} \right]^{T} \qquad (2)$$

Where $\underline{\underline{G}}$ is the open loop TFM, $\underline{\underline{K}}$ is the open loop steady-state gain matrix, and the operator O denotes the Schur product of two matrices

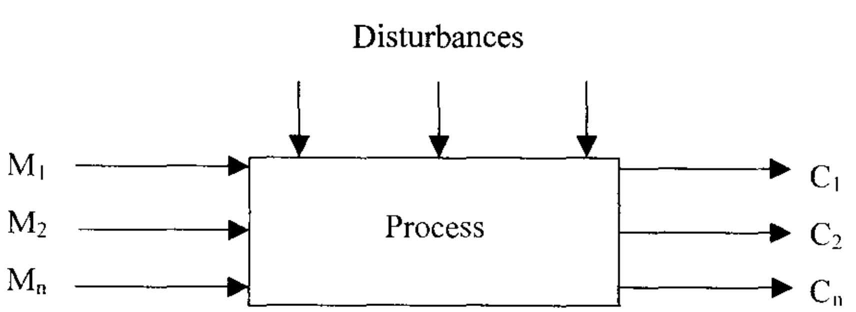

Figure 1. Open Loop Multivariable Process with n-Manipulated Variables M and n-Controlled Variables C

(element by element multiplication). A useful property of the RGA is that the sum of the elements in each row and column is equal to 1. If λ_{ij} were close to zero, then M_j had little effect on C_i; if λ_{ij} were large, then M_j had a significant effect on C_i. In the RGA approach, the recommended pairing is that which corresponds to the relative gain that is positive and closest to 1.

Inverse Nyquist Array (INA)

The INA was the method proposed by Rosenbrock (1969) to indicate the degree of interaction among loops. If the open loop TFM of a process were $\underline{\underline{G}}(s)$, its controller TFM would be $\underline{\underline{Gc}}(s)$ and $\underline{\underline{H}}(i\omega) = \underline{\underline{G}}(i\omega) \times \underline{\underline{Gc}}(i\omega)$, then the INA would be a matrix of the inverse $\underline{\underline{H}}(i\omega)$, which would be $\underline{\underline{H}}^{-1}(i\omega)$.

It is convenient to use the nomenclature

$$\hat{\underline{\underline{H}}}(i\omega) = \underline{\underline{H}}^{-1}(i\omega) \qquad (3)$$

and let the ij*th* element of $\hat{\underline{\underline{H}}}$ be $\hat{h}_{ij}$.

$$\hat{\underline{\underline{H}}} = \begin{bmatrix} \hat{h}_{11} & \hat{h}_{12} & \cdot & \hat{h}_{1N} \\ \hat{h}_{21} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \hat{h}_{N1} & \cdot & \cdot & \hat{h}_{NN} \end{bmatrix} \qquad (4)$$

The INA plots show the plot of diagonal elements of $\hat{\underline{\underline{H}}}(i\omega)$ in all frequencies. Based on the Nyquist stability criterion, these plots should encircle the point (-1,0) counterclockwise if the system were closed loop stable. The sum of the magnitudes of the off-diagonal elements in a corresponding row with $\hat{\underline{\underline{h}}}_{ij}$ would be calculated at several frequencies, and plotted as a radius of a circle with the center at $\hat{\underline{\underline{h}}}_{ij}$ at these frequencies. The so-obtained circles are called *Gershgorin rings*.

If all the off-diagonal elements were zero, the circle would have zero radius and there would be no interaction. Therefore, the bigger the circle, the more interaction would be present in the system. If all the Gershgorin bands encircled the points (-1,0), as shown in Figure 2(a), the system might be *closed loop stable*. On the contrary, if some bands did not encircle the point (-1,0), as shown in Figure 2(b), the system might be *closed loop unstable*.

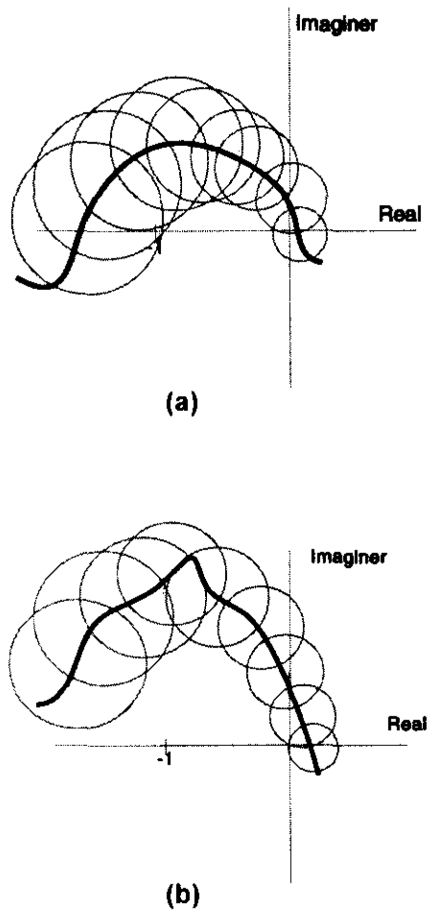

Figure 2. INA Plot for MIMO Systems with Superimposed Gershgorin Bands: (a) Stable and (b) May Be Unstable

Average Dynamic Gain Array (ADGA)

The ADGA was proposed by Gagnepain and Seborg (1982) as an extension of McAvoy's Relative Dynamic Array (1977).

To illustrate this method, consider the following NxN process model:

$$\underline{C}(s) = \underline{G}(s)\underline{M}(s) \qquad (5)$$

where each element of TFM is expressed as an FOPDT model.

$$G_{ij}(s) = \frac{K_{ij} e^{-d_{ij}s}}{T_{ij}s+1} \quad (6)$$

The process is initially at steady state, and a unit step change in M_j occurs at $t=0$. During the time interval $[0,d_{ij}]$, C_i is not affected by M_j, and for the time interval $[d_{ij},q]$, the average dynamic gain D_{ij} between C_j and M_j is calculated as

$$D_{ij} = \frac{1}{\theta-\theta_1}\int_{\theta_1}^{\theta} C_i(t)dt \quad (7)$$

In analogy with the RGA, the matrix of ADGA $\underline{\underline{\mu}}$ can be calculated as

$$\underline{\underline{\mu}} = \underline{\underline{D}} O [\underline{\underline{D}}^{-1}]^T \quad (8)$$

where $\underline{\underline{D}}$ is the matrix with the elements of D_{ij}, and $\theta_1=\min\{\max(d_{ii}),\max(d_{ij})\}$, $\theta=\theta_1+T_M$ with $T_M=\max\{T_{ij}\}$.

Since the ADGA method also normalizes the sum of the average relative dynamic gains in each row and column to 1, in analogy with RGA, it also recommends a pairing that corresponds to the average relative gain that is positive and closest to 1.

RESULTS AND DISCUSSIONS

In this section, several representative models are discussed along with the additional examples studied.

The TFM was chosen to be symmetric in order to tune the controller easily using the Biggest Log-modulus Tuning (BLT) method (Luyben 1986). The elements in the TFM were of the FOPDT model, on which most chemical engineering processes are modeled (Luyben 1997).

The steady-state gains, time constants, and dead times of these elements were varied, and controller pairings were predicted using the RGA, ADGA, and INA methods as given in the table below.

Table 1. Various Values of K, T, and d for a 2x2 FOPDT System

Example	K	T	d
1	1.78	8.16	9
2	1.28	8.16	1
3	1.25	8.16	0.44
4	2.78	1	4
5	1.78	1	0.25
6	1.28	0.14	2.25
7	1.78	0.12	1
8	2.78	0.17	0.25
9	0.36	8.16	9
10	0.56	7.11	1
11	0.78	6.76	0.25
12	0.36	1	4
13	0.56	1	0.25
14	0.78	0.14	2.25
15	0.56	0.17	1
16	0.78	0.12	0.11

where $K = \frac{K_{12}K_{21}}{K_{11}K_{22}}$

$$T = \frac{T_{12}T_{21}}{T_{11}T_{22}}$$

$$d = \frac{d_{12}d_{21}}{d_{11}d_{22}}$$

from Eq. (6).

The transient responses of control configurations suggested by each method were then compared through closed loop simulations using MATLAB™ and SIMULINK™. The investigation started with 2x2 systems and the results were generalized for NxN systems.

The models studied

The models that were studied, along with their relative gain λ_{11} and average relative dynamic gain μ_{11}, are shown in Table 2.

Discussions

The relative gain values, λ_{11}, for examples 1, 2, and 3 show that the recommended pairing

Table 2. The Models Studied

Example 1 $\lambda_{11}=0.36;\mu_{11}=0.68$ $\begin{bmatrix} \frac{-1.5e^{-s}}{7s+1} & \frac{2e^{-3s}}{20s+1} \\ \frac{2e^{-3s}}{20s+1} & \frac{1.5e^{-s}}{7s+1} \end{bmatrix}$	**Example 2** $\lambda_{11}=0.43;\mu_{11}=0.72$ $\begin{bmatrix} \frac{-1.5e^{-3s}}{7s+1} & \frac{1.7e^{-3s}}{20s+1} \\ \frac{1.7e^{-3s}}{20s+1} & \frac{1.5e^{-3s}}{7s+1} \end{bmatrix}$	**Example 3** $\lambda_{11}=0.43;\mu_{11}=0.69$ $\begin{bmatrix} \frac{-1.5e^{-3s}}{5s+1} & \frac{1.7e^{-2s}}{12s+1} \\ \frac{1.7e^{-2s}}{12s+1} & \frac{1.5e^{-3s}}{5s+1} \end{bmatrix}$
Example 4 $\lambda_{11}=0.27;\mu_{11}=0.28$ $\begin{bmatrix} \frac{-1.5e^{-s}}{20s+1} & \frac{2.5e^{-2s}}{20s+1} \\ \frac{2.5e^{-2s}}{20s+1} & \frac{1.5e^{-s}}{20s+1} \end{bmatrix}$	**Example 5** $\lambda_{11}=0.36;\mu_{11}=0.33$ $\begin{bmatrix} \frac{-1.5e^{-2s}}{12s+1} & \frac{2e^{-s}}{12s+1} \\ \frac{2e^{-s}}{12s+1} & \frac{1.5e^{-2s}}{12s+1} \end{bmatrix}$	**Example 6** $\lambda_{11}=0.43;\mu_{11}=0.20$ $\begin{bmatrix} \frac{-1.5e^{-2s}}{16s+1} & \frac{1.7e^{-3s}}{6s+1} \\ \frac{1.7e^{-3s}}{6s+1} & \frac{1.5e^{-2s}}{16s+1} \end{bmatrix}$
Example 7 $\lambda_{11}=0.36;\mu_{11}=0.15$ $\begin{bmatrix} \frac{-1.5e^{-s}}{20s+1} & \frac{2e^{-s}}{7s+1} \\ \frac{2e^{-s}}{7s+1} & \frac{1.5e^{-s}}{20s+1} \end{bmatrix}$	**Example 8** $\lambda_{11}=0.27;\mu_{11}=0.10$ $\begin{bmatrix} \frac{-1.5e^{-2s}}{12s+1} & \frac{2.5e^{-s}}{5s+1} \\ \frac{2.5e^{-s}}{5s+1} & \frac{1.5e^{-2s}}{12s+1} \end{bmatrix}$	**Example 9** $\lambda_{11}=0.73;\mu_{11}=0.91$ $\begin{bmatrix} \frac{-2.5e^{-s}}{7s+1} & \frac{1.5e^{-3s}}{20s+1} \\ \frac{1.5e^{-3s}}{20s+1} & \frac{2.5e^{-s}}{7s+1} \end{bmatrix}$
Example 10 $\lambda_{11}=0.64;\mu_{11}=0.85$ $\begin{bmatrix} \frac{-2e^{-s}}{6s+1} & \frac{1.5e^{-s}}{16s+1} \\ \frac{1.5e^{-s}}{16s+1} & \frac{2e^{-s}}{6s+1} \end{bmatrix}$	**Example 11** $\lambda_{11}=0.57;\mu_{11}=0.77$ $\begin{bmatrix} \frac{-1.7e^{-2s}}{5s+1} & \frac{1.5e^{-s}}{12s+1} \\ \frac{1.5e^{-s}}{12s+1} & \frac{1.7e^{-2s}}{5s+1} \end{bmatrix}$	**Example 12** $\lambda_{11}=0.73;\mu_{11}=0.75$ $\begin{bmatrix} \frac{-2.5e^{-s}}{16s+1} & \frac{1.5e^{-2s}}{16s+1} \\ \frac{1.5e^{-2s}}{16s+1} & \frac{2.5e^{-s}}{16s+1} \end{bmatrix}$
Example 13 $\lambda_{11}=0.64;\mu_{11}=0.62$ $\begin{bmatrix} \frac{-2e^{-2s}}{20s+1} & \frac{1.5e^{-s}}{20s+1} \\ \frac{1.5e^{-s}}{20s+1} & \frac{2e^{-2s}}{20s+1} \end{bmatrix}$	**Example 14** $\lambda_{11}=0.57;\mu_{11}=0.29$ $\begin{bmatrix} \frac{-1.7e^{-2s}}{16s+1} & \frac{1.5e^{-3s}}{6s+1} \\ \frac{1.5e^{-3s}}{6s+1} & \frac{1.7e^{-2s}}{16s+1} \end{bmatrix}$	**Example 15** $\lambda_{11}=0.64;\mu_{11}=0.38$ $\begin{bmatrix} \frac{-2e^{-2s}}{12s+1} & \frac{1.5e^{-2s}}{5s+1} \\ \frac{1.5e^{-2s}}{5s+1} & \frac{2e^{-2s}}{12s+1} \end{bmatrix}$
	Example 16 $\lambda_{11}=0.57;\mu_{11}=0.25$ $\begin{bmatrix} \frac{-1.7e^{-3s}}{20s+1} & \frac{1.5e^{-s}}{7s+1} \\ \frac{1.5e^{-s}}{7s+1} & \frac{1.7e^{-3s}}{20s+1} \end{bmatrix}$	

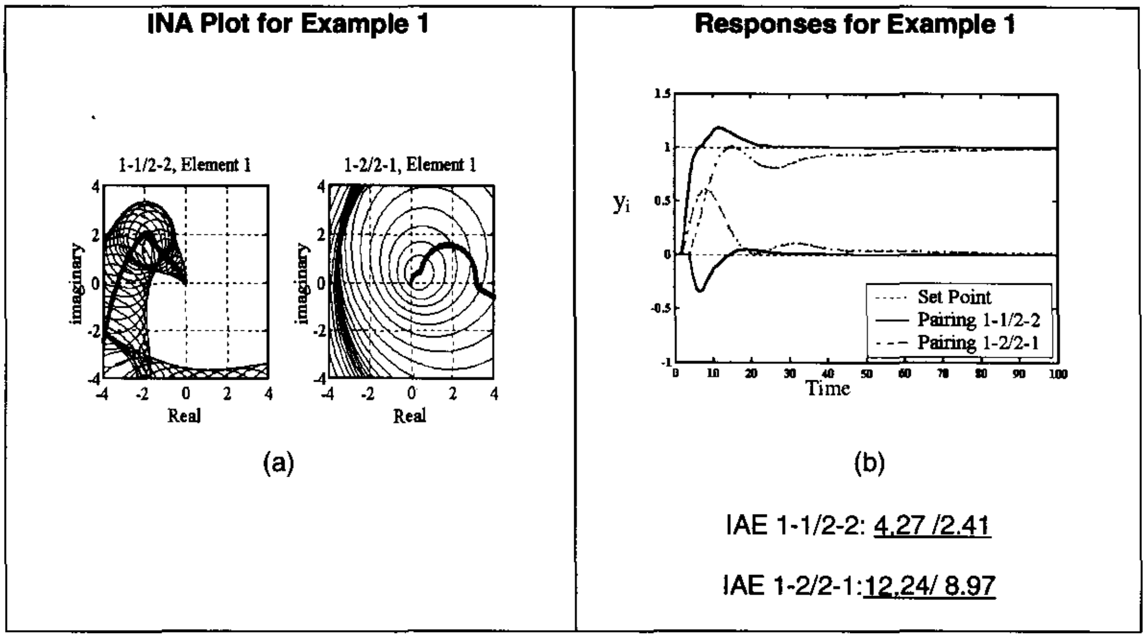

Figure 3. (a) INA Plots and (b) Responses for Example 1

according to the RGA method was 1-2/2-1. On the contrary, the ADGA method suggested 1-1/2-2 pairing for these examples, since the value of μ_{11} was closer to 1 compared to μ_{12}. Note that the sum of μ_{ij} in each row and column equalled to 1.

The INA plot, with Gershgorin bands superimposed, of each possible pairing for model 1 is shown in Figure 3(a). The INA plot for models 2 and 3 are typical. The Gershgorin bands of pairing 1-1/2-2 were smaller than those of pairing 1-2/2-1; therefore, the INA method suggested the same pairing given by the ADGA for models 1, 2, and 3.

To verify which pairing gave the better response, closed loop simulations of each possible pairing for all the models were conducted. The closed loop response of model 1 when subjected to set point change is shown in Figure 3(b).

Pairing 1-2/2-1, which was suggested by the RGA method, gave a poorer response than pairing 1-1/2-2, which was recommended by both the ADGA and INA. This response was seen in the former's slower rise time and settling time as well as bigger integral absolute error (IAE). The same results were obtained for models 2 and 3. Therefore, RGA failed and recommended an incorrect pairing for examples 1, 2, and 3.

For examples 4, 5, 6, 7, and 8, both RGA and ADGA suggested pairing 1-2/2-1, as indicated by the values of λ_{11} and μ_{11} which were closer to zero. The same pairing was also recommended by the INA method; however, the INA plots are not shown in this paper. The closed loop responses for models 4 to 8 also verified that pairing 1-2/2-1 was better than pairing 1-2/2-2, but again the plots are not included here.

For examples 9, 10, 11, 12, and 13, RGA and ADGA both suggested pairing 1-1/2-2. The typical INA plots for these examples, represented here by model 9, can be seen in Figure 4(a). Clearly, the INA method also recommended pairing 1-1/2-2. The closed loop servo responses for model 9 that represented models 10 to 13 also justified that pairing 1-1/2-2 was better than pairing 1-2/2-1. Therefore, the RGA method gave the correct pairing for examples 4 to 13.

For examples 14, 15, and 16, RGA again suggested a different pairing from that recommended by ADGA and INA. The RGA method suggested 1-1/2-2 pairing, whereas ADGA and INA both suggested 1-2/2-1. The closed loop simulations showed again that the RGA failed and gave an incorrect pairing. This case is represented by model 16 in Figure 5.

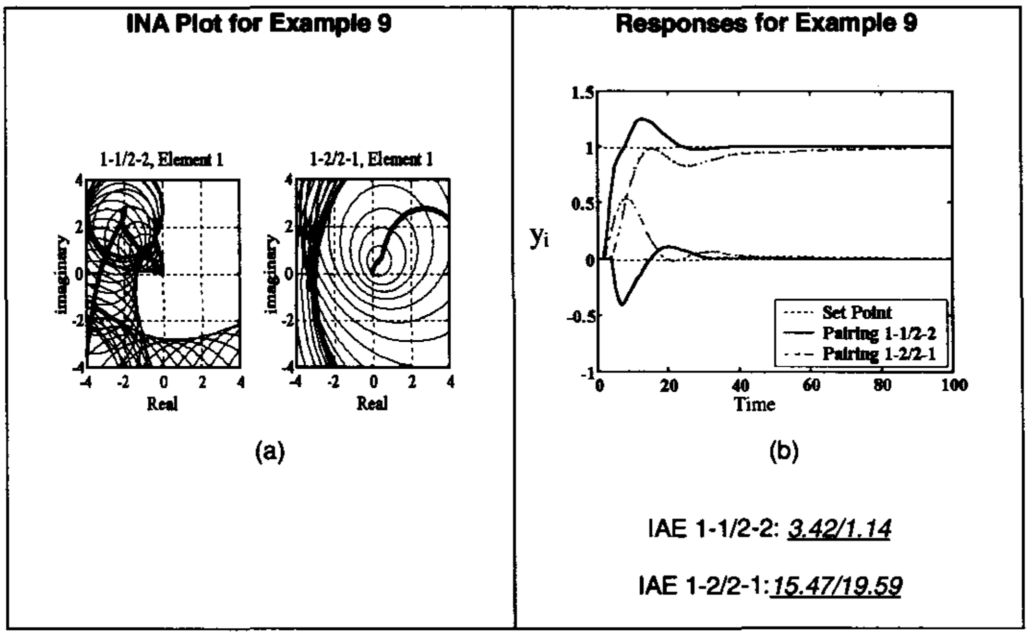

Figure 4. (a) INA Plots and (b) Responses for Example 9

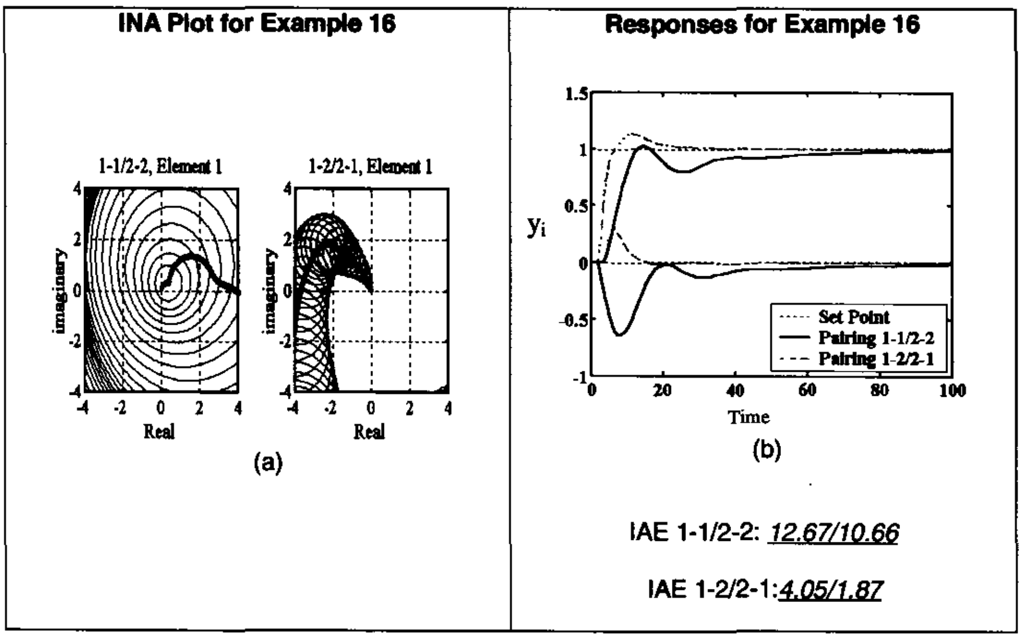

Figure 5. (a) INA Plots and (b) Responses for Example 16

Similar case results for examples 9 to 16 were also obtained by Handogo et al. (2004).

Based on the models studied, it was concluded that RGA's recommendation would always be correct if the pairing elements suggested, compared to other elements in the corresponding rows, had:

(a) bigger steady-state gains and smaller or the same time constants; and,

(b) smaller steady-state gains and bigger or the same time constants.

These conditions were met by models 4 to 13 as discussed earlier.

However, when these conditions were not met, as in the cases of the first three and last three examples, the RGA gave a misleading pairing recommendation. Hence, one should use interaction measurements that include dynamic information, such as ADGA and INA.

Generalization

To make a generalization based on the conclusion earlier, define Q as an open loop TFM with pairing elements arranged as diagonal elements.

$$\underline{\underline{Q}} = \begin{bmatrix} Q_{11} & Q_{12} & . & . & . & Q_{1N} \\ Q_{21} & Q_{22} & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ Q_{N1} & Q_{N2} & . & . & . & Q_{NN} \end{bmatrix} \quad (9)$$

Thus, for 2x2 multiple input multiple output (MIMO) systems with 1-2/2-1 pairing, $Q_{11} = G_{12}$ and $Q_{12} = G_{21}$, whereas for 1-1/2-2 pairing, $Q_{11} = G_{11}$ and $Q_{22} = G_{22}$.

It is clear that to characterize process interactions using the RGA, only an open loop transfer function matrix is needed. The aforementioned conclusion, however, can be generalized using a closed loop TFM approach.

Consider the multiloop control system depicted in Figure 6.

The transfer function relating set point vector $\underline{R}$ and controlled variable vector $\underline{C}$ is:

$$\underline{\underline{C}} = \left[\underline{\underline{I}} + \underline{\underline{QGc}}\right]^{-1} \underline{\underline{QGc}}\underline{R} \quad (10)$$

$$\underline{\underline{C}} = \underline{\underline{G}}_{CL}\ \underline{R} \quad (11)$$

where the closed loop TFM $\underline{\underline{G}}_{CL}$ is:

$$\underline{\underline{G}}_{CL} = \left[\underline{\underline{I}} + \underline{\underline{QGc}}\right]^{-1} \underline{\underline{QGc}} \quad (12)$$

From matrix algebra, the inverse of $[\underline{\underline{I}} + \underline{\underline{QGc}}]$ is defined as:

$$[\underline{\underline{I}} + \underline{\underline{QGc}}]^{-1} = \frac{\left(\underline{\underline{c}}_{[I+QGc]}\right)^{T}}{\det([\underline{\underline{I}} + \underline{\underline{QGc}}])} \quad (13)$$

where $c = \left[\underline{\underline{I}} + \underline{\underline{QG}}_c\right]$ is the matrix of cofactors of $\left[\underline{\underline{I}} + \underline{\underline{QGc}}\right]$.

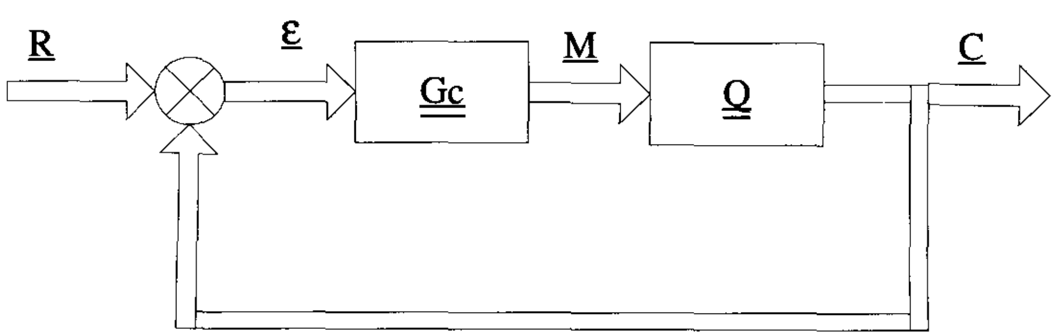

Figure 6. Closed-Loop Multiloop Control System

The ij*th* cofactor of a matrix is defined as

$$c_{ij} = \det(\underline{\underline{M}}_{ij})(-1)^{i+j} \tag{14}$$

where $\underline{M}_{ij}$, the ij*th* *minor of a matrix*, is defined as the matrix left when the i*th* row and j*th* column are removed from the initial matrix.

Thus, Eq. (12) for an NxN multiloop system can be written as:

$$\begin{bmatrix} G_{CL11} & G_{CL12} & \cdot & G_{CL1N} \\ G_{CL12} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ G_{CL1N} & \cdot & \cdot & G_{CLNN} \end{bmatrix} = \frac{\begin{bmatrix} c_{11} & c_{12} & \cdot & c_{1N} \\ c_{21} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{N1} & \cdot & \cdot & c_{NN} \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} & \cdot & Q_{1N} \\ Q_{21} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ Q_{N1} & \cdot & \cdot & Q_{NN} \end{bmatrix} \begin{bmatrix} Gc_1 & 0 & \cdot & 0 \\ 0 & Gc_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & Gc_N \end{bmatrix}}{\det\left(\left[\underline{\underline{I}} + \underline{\underline{Q}}\underline{\underline{Gc}}\right]\right)}$$

$$\begin{bmatrix} G_{CL11} & G_{CL12} & \cdot & G_{CL1N} \\ G_{CL12} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ G_{CL1N} & \cdot & \cdot & G_{CLNN} \end{bmatrix} = \frac{\begin{bmatrix} c_{11} & c_{21} & \cdot & c_{N1} \\ c_{12} & \cdot & \cdot & c_{N2} \\ \cdot & \cdot & \cdot & \cdot \\ c_{1N} & \cdot & \cdot & c_{NN} \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & \cdot & Q_{1N} \\ Q_{21} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ Q_{N1} & \cdot & \cdot & Q_{NN} \end{bmatrix} \begin{bmatrix} Gc_1 & 0 & \cdot & 0 \\ 0 & Gc_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & Gc_N \end{bmatrix}}{\det\left(\left[\underline{\underline{I}} + \underline{\underline{Q}}\underline{\underline{Gc}}\right]\right)}$$

Rewriting this matrix element by element, the following equation is obtained:

$$G_{CL11} = \frac{(c_{11}Q_{11} + c_{21}Q_{21} + \dots\dots\dots + c_{N1}Q_{N1})Gc_1}{\det\left(\left[\underline{\underline{I}} + \underline{\underline{Q}}\underline{\underline{Gc}}\right]\right)}$$

$$G_{CL12} = \frac{(c_{11}Q_{12} + c_{21}Q_{22} + \dots\dots\dots + c_{N1}Q_{N2})Gc_2}{\det\left(\left[\underline{\underline{I}} + \underline{\underline{Q}}\underline{\underline{Gc}}\right]\right)}$$

•

•

$$\boxed{G_{CLij} = \frac{\sum_{k=1}^{N} c_{ki} Q_{kj} Gc_j}{\det\left(\left[\underline{\underline{I}} + \underline{\underline{Q}}\underline{\underline{Gc}}\right]\right)}} \tag{15}$$

For an NxN multiloop system, Eq. (11) can also be written as:

$$\begin{bmatrix} C_1 \\ C_2 \\ . \\ . \\ C_N \end{bmatrix} = \begin{bmatrix} G_{CL11} & G_{CL12} & . & . & G_{CL1N} \\ G_{CL21} & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ .G_{CLN1} & . & . & . & G_{CLNN} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ . \\ . \\ R_N \end{bmatrix}$$

to obtain the following:

$$C_1 = G_{CL11}R_1 + \{G_{CL12}R_2 + G_{CL13}R_3 + \dots\dots\dots + G_{CL1N}R_N\}$$

$$C_2 = G_{CL22}R_2 + \{G_{CL21}R_1 + G_{CL23}R_3 + \dots\dots\dots + G_{CL2N}R_N\}$$

•

•

$$\boxed{C_i = G_{CLii}R_i + \sum_{j=1, j\neq i}^{N} G_{CLij}R_j} \tag{16}$$

Substituting Eq. (15) into Eq. (16), the following is obtained:

$$\boxed{C_i = \sum_{k=1}^{N} \frac{c_{ki}Q_{ki}Gc_i}{\det\left(\left[\underline{I} + \underline{\underline{Q}}\,\underline{\underline{Gc}}\right]\right)} R_i + \sum_{j=1, j\neq i}^{N} \sum_{k=1}^{N} \frac{c_{ki}Q_{kj}Gc_j}{\det\left(\left[\underline{I} + \underline{\underline{Q}}\,\underline{\underline{Gc}}\right]\right)} R_j} \tag{17}$$

Eqs. (16) and (17) illustrate that set point R_i affects the controlled variable C_i through G_{CLii} transmission, and set point R_j, $j\neq i$, affects the controlled variable C_i through G_{CLij} transmission. Thus, R_i affects C_i directly; whereas R_j, $j\neq i$, affects C_i by interaction. Therefore, G_{CLii} is called *direct transmission* and G_{CLij} is called *interaction transmission*.

In designing the multiloop control system, the goal was to build a control configuration with *minimum interaction*; that is, R_j could be changed with no significant effect on C_i, $j\neq i$.

Therefore, the correct pairing should make the magnitude ratio of direct transmission G_{CLii} greater than the magnitude ratio of interaction transmission G_{CLij}, $j\neq 1$.

The following section will show that the magnitude ratio of C_{Lii} relative to the magnitude ratio of G_{CLij} can be approximated by comparing the magnitude ratios of Q_{ii} and Q_{ij}.

As an illustration, for 2x2 systems where

$$\left[\underline{I} + \underline{\underline{Q}}\,\underline{\underline{Gc}}\right] = \begin{bmatrix} 1 + Q_{11}Gc_1 & Q_{12}Gc_2 \\ Q_{21}Gc_1 & 1 + Q_{22}Gc_2 \end{bmatrix} \tag{18}$$

from Eq. (15) the following is obtained:

$$G_{CL11} = \frac{\sum_{k=1}^{2} c_{ki} Q_{kj} Gc_j}{\det\left(\left[\underline{I} + \underline{\underline{QGc}}\right]\right)} = \frac{c_{11}Q_{11}Gc_1 + c_{21}Q_{21}Gc_1}{\det\left(\left[\underline{I} + \underline{\underline{QGc}}\right]\right)} \tag{19}$$

Using the definition of *cofactor*, c_{11} and c_{21} of matrix 18 are

$$c_{11} = 1 + Q_{22}G_{C2} \quad \text{and} \quad c_{21} = -Q_{12}G_{C2} \tag{20}$$

while the determinant of matrix 18 is

$$\begin{aligned}\det([\underline{I} + \underline{\underline{QGc}}]) &= (1 + Q_{11}Gc_1)(1 + Q_{22}Gc_2) - (Q_{12}Gc_2)(Q_{21}Gc_1) \\ &= 1 + Q_{22}Gc_2 + Q_{11}Gc_1 + (Q_{11}Q_{22} - Q_{12}Q_{21})Gc_1Gc_2\end{aligned} \tag{21}$$

Substituting equations (20) and (21) into Eq. (19) gives

$$G_{CL11} = \frac{Q_{11}Gc_1 + Q_{11}Q_{22}Gc_1Gc_2 - Q_{12}Q_{21}Gc_1Gc_2}{1 + Q_{22}Gc_2 + Q_{11}Gc_1 + (Q_{11}Q_{22} - Q_{12}Q_{21})Gc_1Gc_2} \tag{22}$$

To obtain a clearer relationship between G_{CL11} and Q_{11}, long division can be done on Eq. (22):

$$\begin{array}{r|l} & Gc_1Q_{11} - Gc_1Q_{21}Gc_2Q_{12} + \ldots\ldots\ldots\ldots \\ \hline \begin{pmatrix}1 + Q_{22}Gc_2 + Q_{11}Gc_1 + \\ (Q_{11}Q_{22} - Q_{12}Q_{21})Gc_1Gc_2\end{pmatrix} & Q_{11}Gc_1 + Q_{11}Q_{22}Gc_1Gc_2 - Q_{12}Q_{21}Gc_1Gc_2 \\ & \underline{Q_{11}Gc_1 + Q_{11}Q_{22}Gc_1Gc_2 + (Q_{11}Gc_1)^2 + Gc_1^{\,2}Q_{11}Gc_2(Q_{11}Q_{22} - Q_{21}Q_{12})} \\ & - Gc_1Q_{21}Gc_2Q_{12} - (Q_{11}Gc_1)^2 - Gc_1^{\,2}Q_{11}Gc_2(Q_{11}Q_{22} - Q_{21}Q_{12}) \\ & \underline{- Gc1Q21Gc2Q12 - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots} \\ & \ldots\ldots\ldots\ldots \end{array}$$

Finally, the following expression is obtained

$$G_{CL11} = G_{C1}Q_{11} - G_{C1}Q_{21}G_{C2}Q_{12} + \ldots\ldots\ldots \tag{23}$$

Repeating the steps from Eq. (19) to Eq. (23) for G_{CL12}, G_{CL21}, *and* G_{CL22} results in:

$$\begin{aligned} G_{CL12} &= G_{C2}Q_{12} + \ldots\ldots\ldots \\ G_{CL21} &= G_{C1}Q_{21} + \ldots\ldots\ldots \\ G_{CL22} &= G_{C2}Q_{22} - G_{C2}Q_{12}G_{C1}Q_{21} + \ldots\ldots\ldots \end{aligned} \tag{24}$$

The physical meaning of the infinite number of terms in Eqs. (23) and (24) can be understood by recalling that the change in M_1 affected loop 2 through Q_{21}, then the signal passed loop 2 before returning to loop 1 via Q_{12}, until the new steady-state condition was achieved. Since the diagonal elements of $\underline{\underline{Q}}(s)$ are typically larger than the off-diagonal elements, it is expected that the more Q_{ij}, $j \neq i$, that the signal passes through, the smaller the transmission will be. Therefore, the magnitude ratio of G_{CLii} can be approximated by the magnitude ratio of Q_{ii} solely. Consecutively, the magnitude ratio of G_{CLii} relative to G_{CLij}, $j \neq i$, can be approximated by comparing the magnitude ratio of Q_{ii} and Q_{ij}.

Hence, when the RGA suggests G_{ij} as a pairing element, that is to be placed as a Q_{ii} element in the $\underline{\underline{Q}}(s)$ matrix, this prediction will be correct if the magnitude ratio of the recommended G_{ij} is greater than the magnitude ratio of the other elements in the corresponding row.

For the FOPDT model

$$G_{ij}(s) = \frac{K_{ij}e^{-d_{ij}s}}{T_{ij}s+1}$$

the magnitude ratio of G_{ij} is

$$|G_{ij}(i\omega)| = \left|\frac{K_{ij}e^{-d_{ij}(i\omega)}}{T_{ij}(i\omega)+1}\right| = \frac{|Kij|}{\sqrt{1+\omega^2 T_{ij}^{\ 2}}} \qquad (25)$$

From Eq. (25), it is understood that G_{ij} has a greater magnitude ratio than the other elements in the corresponding row if its (a) gain K_{ij} is greater and (b) its time constant T_{ij} is equal or smaller than the gains and time constants of the other elements in the row.

If these conditions were met by the elements recommended as pairing by the RGA, it could be expected that RGA's prediction would be correct. Otherwise, one should use an interaction measurement that includes both static and dynamic process behaviors, such as ADGA and INA.

To see the applicability of that conclusion to systems with a higher order, two modified 3x3 systems based on Gagnepain and Seborg (1982) are presented below.

Example A

$$\underline{\underline{G}}(s) = \begin{bmatrix} \dfrac{-2e^{-s}}{s+1} & \dfrac{1.5e^{-s}}{5s+1} & \dfrac{e^{-s}}{10s+1} \\ \dfrac{1.5e^{-s}}{5s+1} & \dfrac{-e^{-s}}{10s+1} & \dfrac{2e^{-s}}{s+1} \\ \dfrac{e^{-s}}{10s+1} & \dfrac{2e^{-s}}{s+1} & \dfrac{1.5e^{-s}}{5s+1} \end{bmatrix}$$

Example B

$$\underline{\underline{G}}(s) = \begin{bmatrix} \dfrac{-2e^{-s}}{10s+1} & \dfrac{1.5e^{-s}}{5s+1} & \dfrac{e^{-s}}{1s+1} \\ \dfrac{1.5e^{-s}}{5s+1} & \dfrac{-e^{-s}}{1s+1} & \dfrac{2e^{-s}}{10s+1} \\ \dfrac{e^{-s}}{1s+1} & \dfrac{2e^{-s}}{10s+1} & \dfrac{1.5e^{-s}}{5s+1} \end{bmatrix}$$

The RGA for model A is

$$\underline{\underline{\lambda}} = \begin{bmatrix} 0.7521 & -0.0256 & 0.2735 \\ -0.0256 & 0.2735 & 0.7521 \\ 0.2735 & 0.7521 & -0.0256 \end{bmatrix}$$

while the ADGA for model A is

$$\underline{\underline{\mu}} = \begin{bmatrix} 0.9195 & -0.0077 & 0.0882 \\ -0.0077 & 0.0882 & 0.9195 \\ 0.0882 & 0.9195 & -0.0077 \end{bmatrix}$$

Both RGA and ADGA recommended the pairing 1-1/2-3/3-2.

The INA plot and closed-loop responses plot for model A on the following page compares the performance of the pairing 1-1/2-3/3-2 with that of 1-3/2-1/3-2, which is another possible pairing.

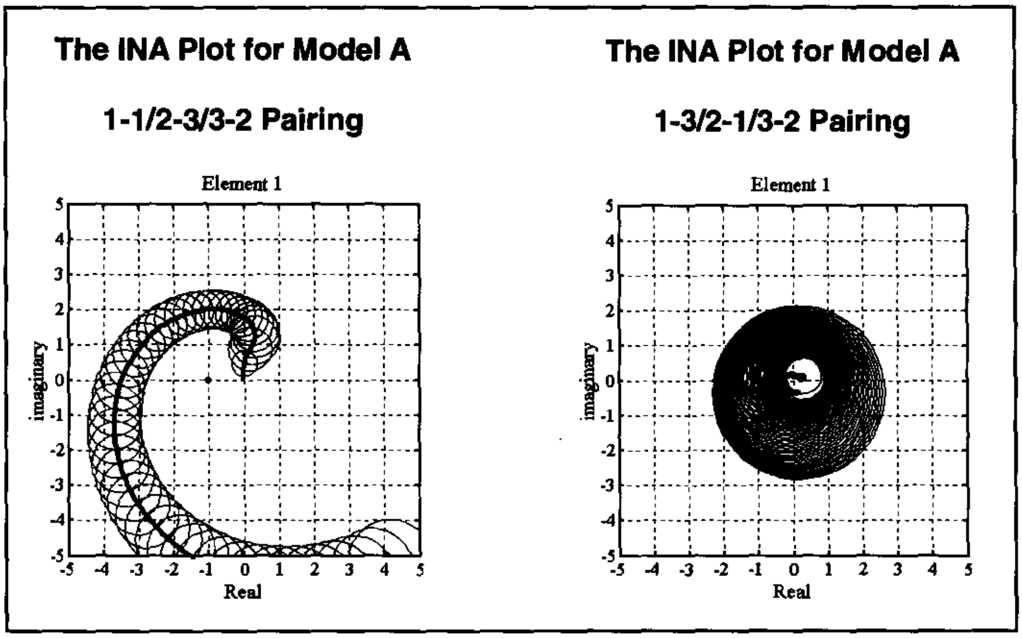

Figure 7. The INA Plot for Model A with Gershgorin Band Superimposed

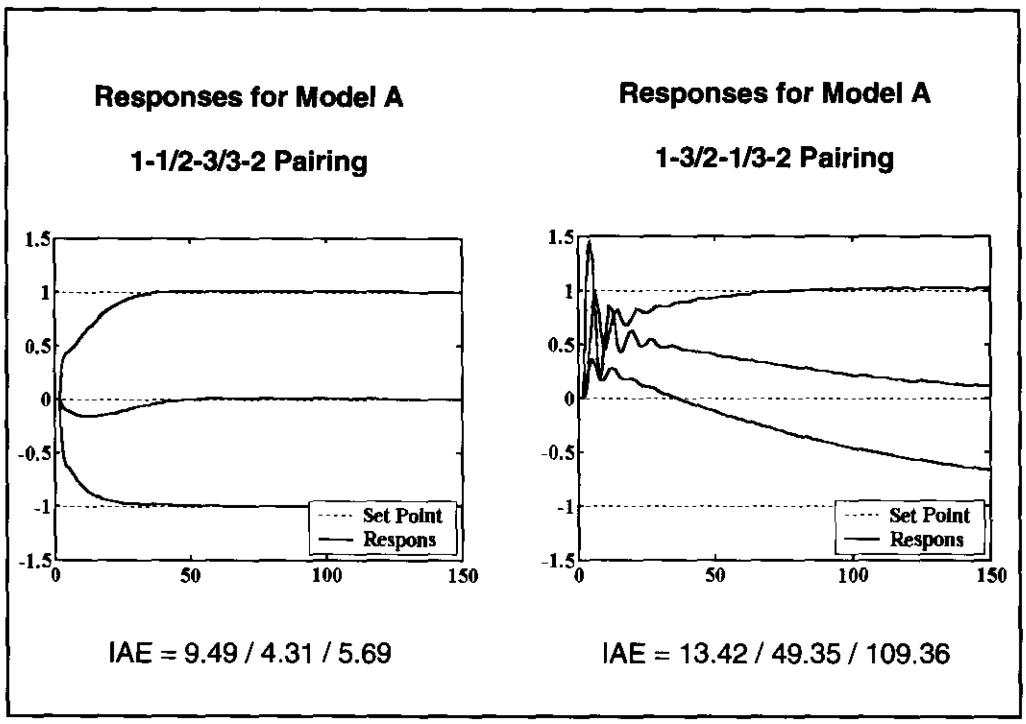

Figure 8. Closed-Loop Responses for Model A

Figures 7 and 8 show that 1-1/2-3/3-2 pairing is the correct configuration. Thus, RGA has recommended the proper pairing. Note that for model A, the elements suggested as pairing elements by RGA meet the conditions mentioned above.

The RGA for model B is

$$\underline{\underline{\lambda}} = \begin{bmatrix} 0.7521 & -0.0256 & 0.2735 \\ -0.0256 & 0.2735 & 0.7521 \\ 0.2735 & 0.7521 & -0.0256 \end{bmatrix}$$

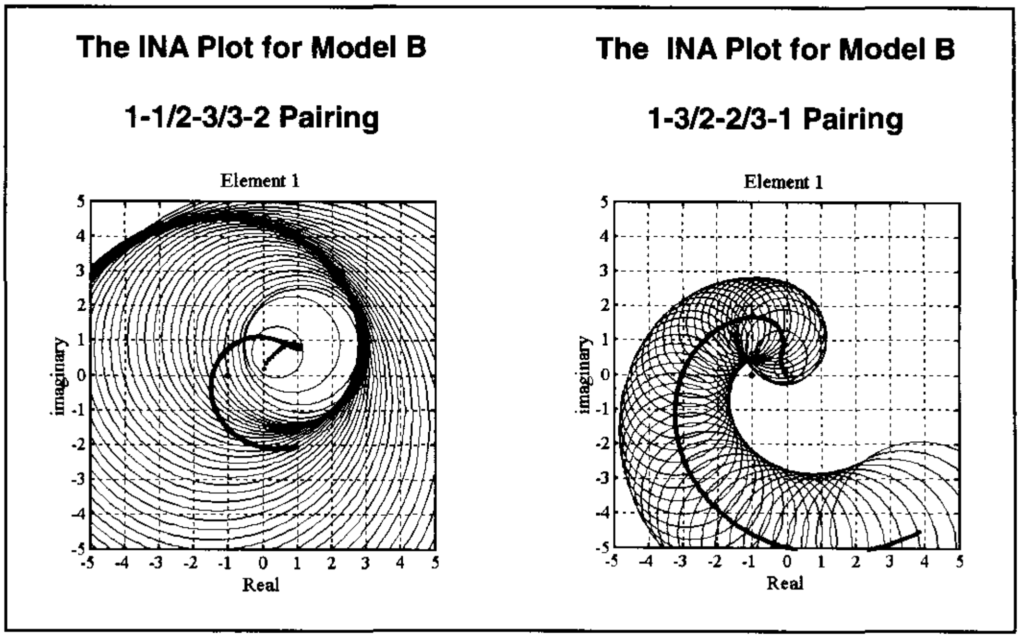

Figure 9. TNA Plot for Model B with Gershgorin Band Superimposed

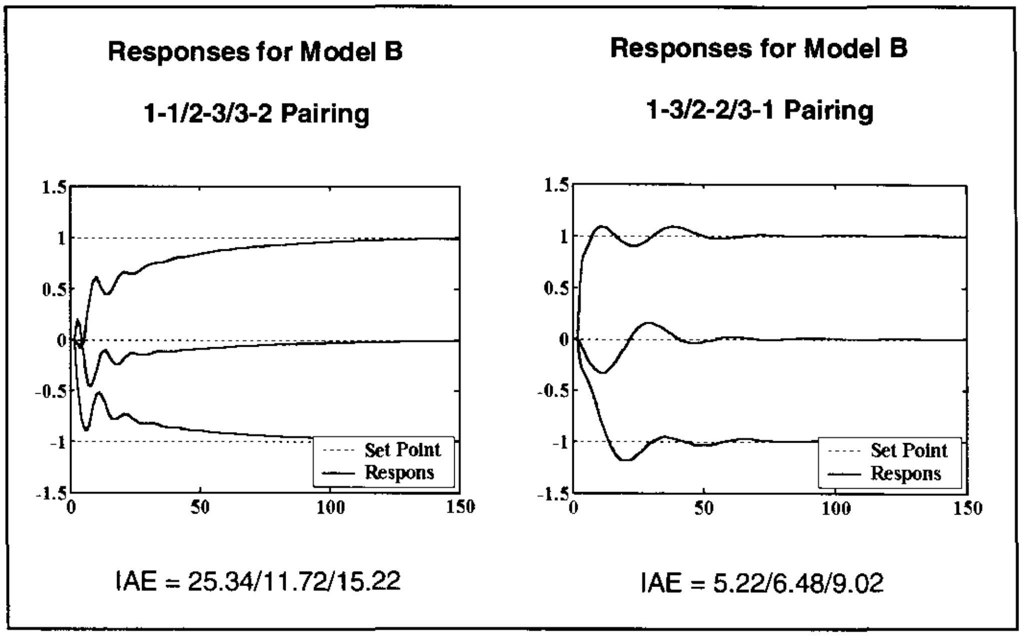

Figure 10. Closed Loop Responses for Model B

whereas the ADGA for model B is

$$\underline{\underline{\mu}} = \begin{bmatrix} 0.4370 & -0.0243 & 0.5873 \\ -0.0243 & 0.5873 & 0.4370 \\ 0.5873 & 0.4370 & -0.0243 \end{bmatrix}$$

Now, the RGA suggested a different pairing from the ADGA. RGA predicted 1-1/2-3/3-2 pairing, whereas ADGA predicted 1-3/2-2/3-1 pairing. The INA plot in Figure 9 also suggested 1-3/2-2/3-1 pairing. To verify which pairing would be best, closed loop simulation should be conducted to see their respective responses.

Figure 10 shows that 1-3/2-2/3-1 pairing gives a better response than 1-1/2-3/3-2 pairing. Hence, in this case, RGA has given another misleading recommendation. Upon inspection of the elements suggested as pairing by RGA, it was revealed that the elements did not meet the conditions mentioned, since both their gains and time constants were bigger compared to those of other elements in the corresponding rows. Therefore, the RGA method suggested an improper pairing.

CONCLUSIONS

The present study compared the steady-state interaction measurement Relative Gain Array with two dynamic interaction measurements, Average Dynamic Gain Array and Inverse Nyquist Array.

It was concluded that RGA could be expected to recommend the correct pairing if the pairing elements it suggested had (a) bigger steady-state gains and (b) smaller or the same time constants than the other elements in corresponding rows.

If these conditions were not met, the RGA would give a misleading recommendation on pairing controller and an interaction measurement that includes both the dynamic parameter and the static parameter behaviors in a process, such as the ADGA or the INA, should be used instead.

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